MAX IV 3 GeV Storage Ring Lattice
“State of Affairs”

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The Current Achromat Design

RING: 20 achromats
- \( L = 527.76 \text{ m} \)
- 19 available straights (5m)
- 40 short straights (1375 mm)
- \( \varepsilon_0 = 0.336 \text{ nm rad} \)
- \( \nu_x = 42.20 \)
- \( \nu_y = 14.28 \)
- \( \eta^* = 0.0 \text{ m} \) Dispersion-free

- original \( \beta_x^* \), \( \beta_y^* \) restored:
  - \( \beta_x^* = 9.33 \text{ m} \rightarrow \sigma_x = 56 \mu\text{m} \)
  - \( \beta_y^* = 4.82 \text{ m} \rightarrow \sigma_y = 2.2 \mu\text{m} \)

- \( \xi_{x,0} = -49.5 \)
- \( \xi_{y,0} = -42.7 \)

2 Split QF Families
3 Separate SF Families
The Current Achromat Design: Dipoles

- \( \text{DIP} = (\text{DIP}_m) \times (-\text{DIP}_m) \)
- soft ends
- \( \varphi = 3^\circ \)
- \( L = 1.3 \text{ m} \)
- \( B_0 = 0.55 \text{ T (max)} \)
- Gradient dipoles \( \rightarrow \) QD: \( k_{\text{max}} = -9.14 \text{ T/m} \)
The Current Achromat Design: Quadrupoles

QFend / QDend
- $L = 20 \text{ cm}$
- $k = 39.5 \text{ T/m}, -23.7 \text{ T/m}$

QFm / QF
- $L = 15 \text{ cm}$
- $k = 37.9 \text{ T/m}, 40.1 \text{ T/m}$

QD (in bends)
- $k = -9.14 \text{ T/m}$
The Current Achromat Design: Sextupoles

SDend / SD
- L = 10 cm
- m = -1283 T/m², -1182 T/m²

SFm / SFo / SFi
- L = 10 cm
- m = 1926 T/m², 1341 T/m², 2130 T/m²
Lattice Performance (1)

Dynamic Aperture (bare lattice, 2.5 DOF)

Required Aperture
Lattice Performance (2)

Lattice Momentum Acceptance?

Possible Limitations!
Improving the 3 GeV Lattice with Octupoles
(in collaboration with Andreas Streun, SLS/PSI)
The Problem (1)

- MAX IV has ultra-low emittance → need sufficient dynamic aperture (DA) and momentum acceptance (MA) for efficient capture and sufficient lifetime
- Tune “footprint” is too large → we cross too many resonance lines → bad for DA and MA

![Graph showing tunes in sext. and skew sext couplings with ΔA_x, ΔA_y, WP, Δ=-6% and Δ=+6% parameters]
The Problem (2)

- Source: Sextupoles (required to correct negative natural chromaticity) drive higher-order terms \(\rightarrow\) strong growth of tune shift with energy (ChrTS) and with amplitude (ADTS)
Octupole Hamiltonian & Driving Terms (I)

- Octupole Hamiltonian

\[
H_4 = \frac{b_4}{4} \left[ x^4 - 6x^2y^2 + y^4 \right] \left( 1 - \delta + \delta^2 + O(\delta^3) \right)
\]

\[b_4: \text{int. oct. strength in } [\text{T/m}^3]\]

\[\delta = \Delta p/p\]

- Betatron motion on dispersive orbits

\[
x = x_\beta + \eta \delta
\]

\[\rightarrow H_4 = \frac{b_4}{4} \left[ \left( x_\beta^4 - 6x_\beta^2y^2 + y^4 \right) + \delta \left( -x_\beta^4 + 6x_\beta^2y^2 - y^4 + 4x_\beta^3\eta - 12x_\beta y^2\eta \right) + \delta^2 \left( x_\beta^4 - 6x_\beta^2y^2 + y^4 - 4x_\beta^3\eta + 12x_\beta y^2\eta + 6x_\beta^2\eta^2 - 6y^2\eta^2 \right) + O(\delta^3) \right]
\]

- Resonance basis (Johan Bengtsson)

\[
h_{x,y}^\pm = \sqrt{2J_{x,y}} \sqrt{\beta_{x,y}} \times e^{\pm i\varphi_{x,y}} \quad \rightarrow \quad x_\beta = \frac{1}{2} (h_{x+} + h_{x-})
\]
Octupole Hamiltonian & Driving Terms (2)

- Express octupole Hamiltonian in resonance basis

\[ H_4 = \frac{b_4}{4} \frac{1}{16} (h_{x+}^4 + 4h_{x+}^3h_{x-} + 6h_{x+}^2h_{x-}^2 + 4h_{x+}h_{x-}^3 + h_{x-}^4) \]

+... in total 83 terms to 2nd order in \( \delta \)

- Gather octupole terms by phase

\[ H_4 = \sum h_{jklmn} \]

\[ = \sum \delta^n h_{x+}^j h_{x-}^k h_{y+}^l h_{y-}^m \]

\[ = \sum \delta^n (2J_x/\beta_x)^{j+k} (2J_y/\beta_y)^{l+m} e^{i[(j-k)\varphi_x + (l-m)\varphi_y]}. \]

- Inspect first-order octupole terms

\[ (j + k + l + m + n) = 4 \]
Octupole Hamiltonian & Driving Terms (3)

- First order octupole terms with phases $\rightarrow$ drive resonances

\[
\begin{align*}
    h_{40000} &= \frac{b_4}{64} \beta_x^2 (2J_x)^2 e^{i4\varphi_x} \quad \text{drives } 4Q_x \\
    h_{31000} &= \frac{b_4}{16} \beta_x^2 (2J_x)^2 e^{i2\varphi_x} \quad \text{drives } 2Q_x \\
    h_{13000} &= \frac{b_4}{16} \beta_x^2 (2J_x)^2 e^{-i2\varphi_x} \quad \text{drives } 2Q_x \\
    h_{04000} &= \frac{b_4}{64} \beta_x^2 (2J_x)^2 e^{-i4\varphi_x} \quad \text{drives } 4Q_x \\
    h_{20200} &= -\frac{3b_4}{32} \beta_x \beta_y (2J_x)(2J_y) e^{i(2\varphi_x+2\varphi_y)} \quad \text{drives } 2Q_x + 2Q_y \\
    h_{20110} &= -\frac{3b_4}{16} \beta_x \beta_y (2J_x)(2J_y) e^{i2\varphi_x} \quad \text{drives } 2Q_x \\
    h_{20020} &= -\frac{3b_4}{32} \beta_x \beta_y (2J_x)(2J_y) e^{i(2\varphi_x-2\varphi_y)} \quad \text{drives } 2Q_x - 2Q_y \\
    h_{11200} &= -\frac{3b_4}{16} \beta_x \beta_y (2J_x)(2J_y) e^{i2\varphi_y} \quad \text{drives } 2Q_y \\
    h_{11020} &= h_{11200}^* \quad \text{drives } 2Q_y \\
    h_{02200} &= -\frac{3b_4}{32} \beta_x \beta_y (2J_x)(2J_y) e^{-i(2\varphi_x-2\varphi_y)} \quad \text{drives } 2Q_x - 2Q_y \\
    h_{02110} &= -\frac{3b_4}{16} \beta_x \beta_y (2J_x)(2J_y) e^{-i2\varphi_x} \quad \text{drives } 2Q_x \\
    h_{02020} &= h_{20200}^* \quad \text{drives } 2Q_x \\
    h_{00400} &= \frac{b_4}{64} \beta_y^2 (2J_y)^2 e^{i4\varphi_y} \quad \text{drives } 4Q_y
\end{align*}
\]

... and many more (30 in total)
Octupole Hamiltonian & Driving Terms (4)

• Some first order octupole terms carry no phase → **tune shifts**

\[
\begin{align*}
   h_{22000} &= \frac{3}{8} b_4 \beta_x^2 J_x^2 \\
   h_{11110} &= -\frac{3}{2} b_4 \beta_x \beta_y 2 J_x J_y \\
   h_{00220} &= \frac{3}{8} b_4 \beta_y^2 J_y^2 \\
   h_{11002} &= \frac{6}{4} b_4 \eta^2 \delta^2 \beta_x J_x \\
   h_{00112} &= -\frac{6}{4} b_4 \eta^2 \delta^2 \beta_y J_y
\end{align*}
\]

• Recall that \(2\pi \nu\) and \(J\) are conjugate action-angle variables

\[
\nu_{x,y} = \frac{1}{2\pi} \frac{\partial H}{\partial J_{x,y}}
\]
Octupole Hamiltonian & Driving Terms (5)

\[ \nu_{x,y} = \frac{1}{2\pi} \frac{\partial H}{\partial J_{x,y}} \quad \rightarrow \quad \frac{\partial \Delta \nu_{x,y}}{\partial J_{x,y}} = \frac{1}{2\pi} \frac{\partial^2 H}{\partial J_{x,y}^2} \]

• 3 driving terms for ADTS

\[ \Rightarrow \frac{\partial \Delta \nu_x}{\partial J_x} = \frac{3}{8\pi} b_4 \beta_x^2 \]
\[ \Rightarrow \frac{\partial \Delta \nu_y}{\partial J_y} = \frac{3}{8\pi} b_4 \beta_y^2 \]
\[ \Rightarrow \frac{\partial \Delta \nu_x}{\partial J_y} = \frac{\partial \Delta \nu_y}{\partial J_x} = -\frac{3}{4\pi} b_4 \beta_x \beta_y \]
Octupole Hamiltonian & Driving Terms (6)

\[ \nu_{x,y} = \frac{1}{2\pi} \frac{\partial H}{\partial J_{x,y}} \rightarrow \frac{\partial^2 \Delta \nu_{x,y}}{\partial \delta^2} = \frac{1}{2\pi} \frac{\partial^2}{\partial \delta^2} \frac{\partial H}{\partial J_{x,y}} \]

- 2 driving terms for ChrTS (quadratic chromaticity)

\[ \xi^{(2)}_{x,y} = \frac{1}{2} \frac{\partial^2 \Delta \nu_{x,y}}{\partial \delta^2} = \frac{1}{4\pi} \frac{\partial^2}{\partial \delta^2} \frac{\partial H}{\partial J_{x,y}} \]

\[ \Rightarrow \xi^{(2)}_{x,y} = \pm \frac{3}{4\pi} b_4 \eta^2 \beta_{x,y} \]
How Octupoles Can be Used to Solve Problems (1)

- Matrix formalism for 5 higher-order driving terms and n octupole families

\[
\begin{pmatrix}
\frac{\partial \nu_x}{\partial J_x} \\
\frac{\partial \nu_x}{\partial J_y} \\
\frac{\partial \nu_y}{\partial J_x} \\
\frac{\partial^2 \nu_x}{\partial \delta^2} \\
\frac{\partial^2 \nu_y}{\partial \delta^2}
\end{pmatrix}
= \mathcal{M}_{5 \times n}
\begin{pmatrix}
(b_4l)_1 \\
\vdots \\
(b_4l)_n
\end{pmatrix}
\]

5 known expressions evaluated at n octupole locations ($\beta_x$, $\beta_y$, $\eta$)

- From OPA we know what the 5 higher-order driving terms generated by the sextupole are...

Reverse the problem: what octupole strengths $(b_4l)_j$ are required to exactly cancel these 5 terms?
How Octupoles Can be Used to Solve Problems (2)

- Inversion or SVD → use octupoles to cancel higher-order terms generated by sextupoles

\[ \mathcal{M}_{5 \times n}^{-1} \times (-1) \times \begin{pmatrix} \partial \nu_x / \partial J_x \\ \vdots \\ \partial^2 \nu_y / \partial \delta^2 \end{pmatrix} = \begin{pmatrix} (b_{4l})_1 \\ \vdots \\ (b_{4l})_n \end{pmatrix} \]

- If \( n = 5 \) → matrix inversion → perfect cancellation
- If \( n > 5 \) → SVD required → perfect cancellation, minimize req. oct. strength
- If \( n < 5 \) → SVD required → minimize driving terms
- If no chromatic octupoles available → reduce to 3-dim. harmonic problem
Octupoles in the MAX IV Lattice (1)

- Studied different layouts with up to six octupole families
- Chromatic octupoles appear to be detrimental (higher-order chromatic terms)
- Harmonic octupoles at right locations are very efficient (5cm length!)
Octupoles in the MAX IV Lattice (2)

- Three weak and compact families are sufficient for near-perfect cancellation of ADTS while retaining sufficiently small ChrTS

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<th>Target</th>
<th>Value</th>
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</tr>
<tr>
<td>CrY lin</td>
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<tr>
<td>Qx H21000</td>
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<tr>
<td>3Qx H300000</td>
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<td>2Qy H00201</td>
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</tbody>
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- Table showing values for different targets and parameters.
Octupoles in the MAX IV Lattice (3)

- Within required DA we cross not a single regular resonance below 4th order
- MA of the lattice better than ±5%
Octupoles in the MAX IV Lattice (4)