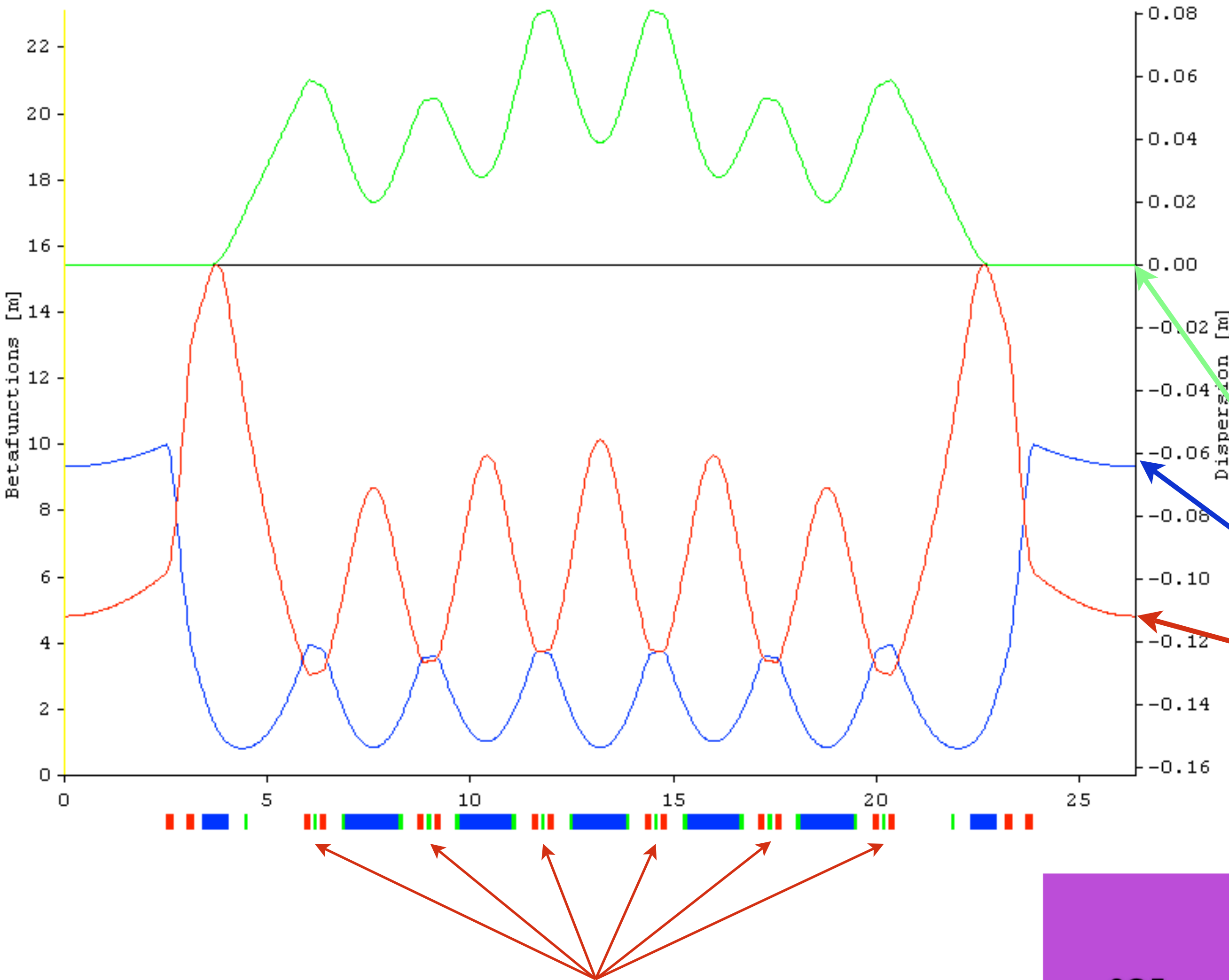
MAX IV 3 GeV Storage Ring Lattice
“State of Affairs”

simon.leemann@maxlab.lu.se

The Current Achromat Design



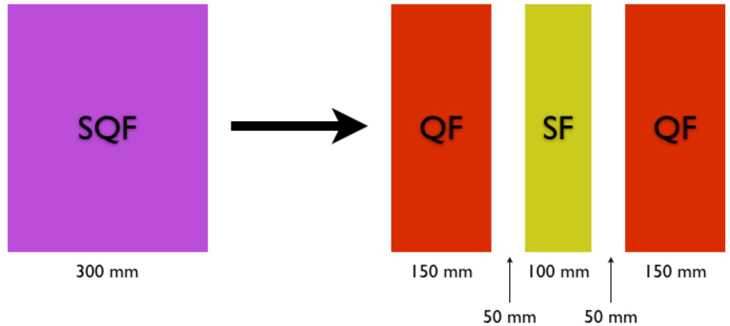
2 Split QF Families
3 Separate SF Families

RING: 20 achromats

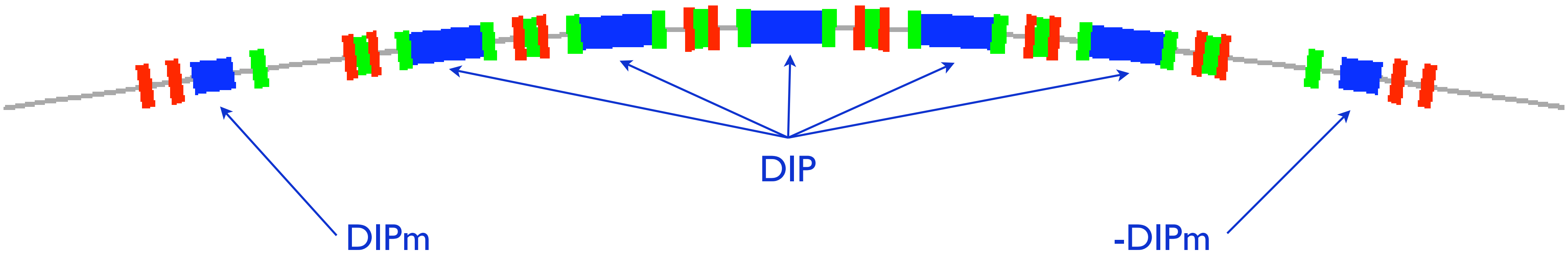
- L = 527.76 m
- 19 available straights (5m)
- 40 short straights (1375 mm)
- ε_0 = 0.336 nm rad
- ν_x = 42.20
- ν_y = 14.28

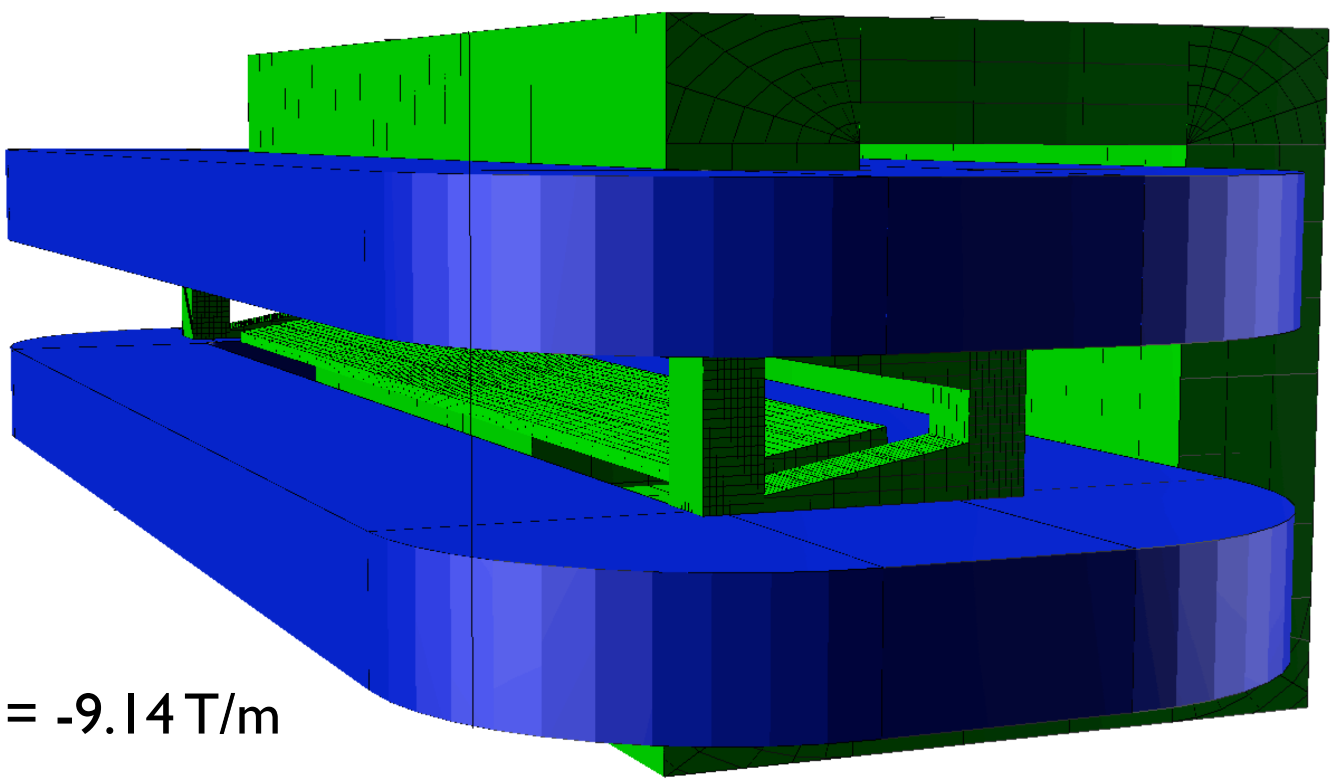
} New WP

- η^* = 0.0 m Dispersion-free
- original β_x^*, β_y^* restored:
  - β_x^* = 9.33 m → σ_x=56 μm
  - β_y^* = 4.82 m → σ_y=2.2 μm
- $\xi_{x,0}$ = -49.5
- $\xi_{y,0}$ = -42.7



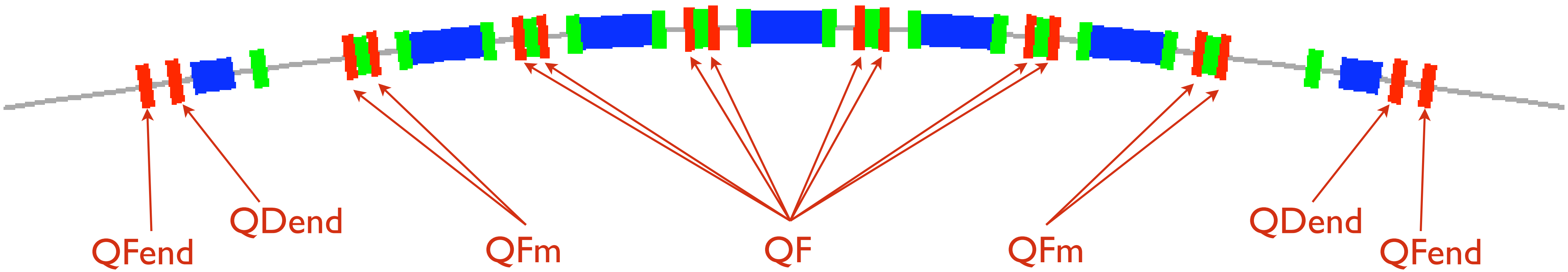
The Current Achromat Design: Dipoles





- DIP = (DIPm) × (-DIPm)
- soft ends
- $\varphi = 3°$
- L = 1.3 m
- $B_0 = 0.55$ T (max)
- Gradient dipoles → QD: $k_{max} = -9.14$ T/m

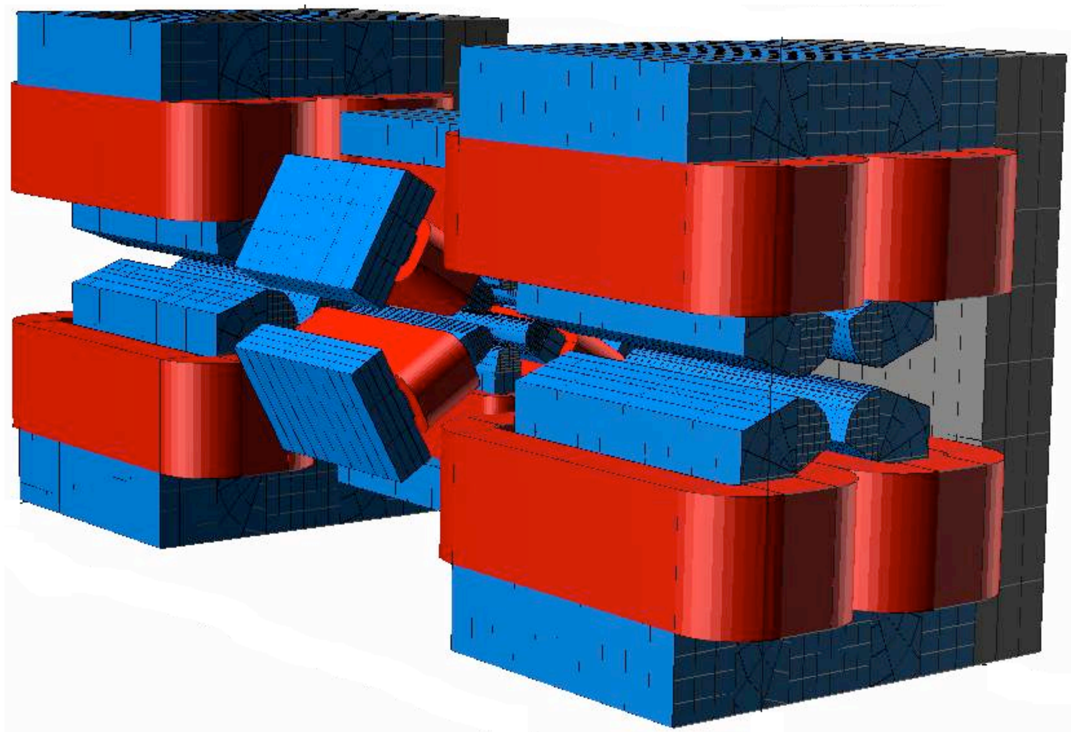
The Current Achromat Design: Quadrupoles



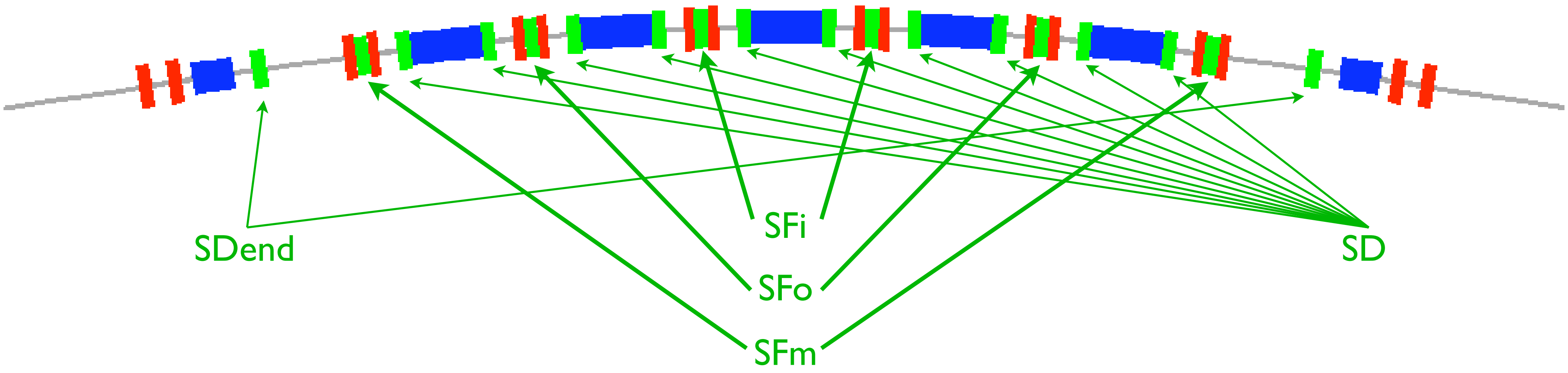
QFend / QDend
- L = 20 cm
- k = 39.5 T/m, -23.7 T/m

QFm / QF
- L = 15 cm
- k = 37.9 T/m, 40.1 T/m

QD (in bends)
- k = -9.14 T/m



The Current Achromat Design: Sextupoles

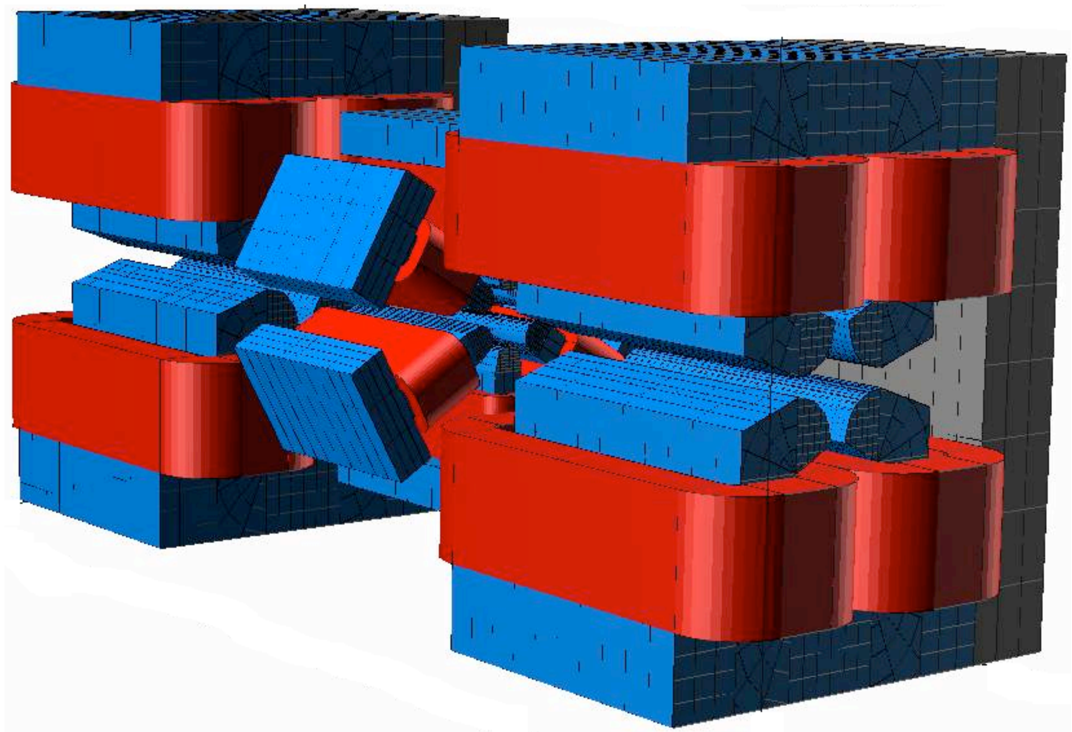


SDend / SD

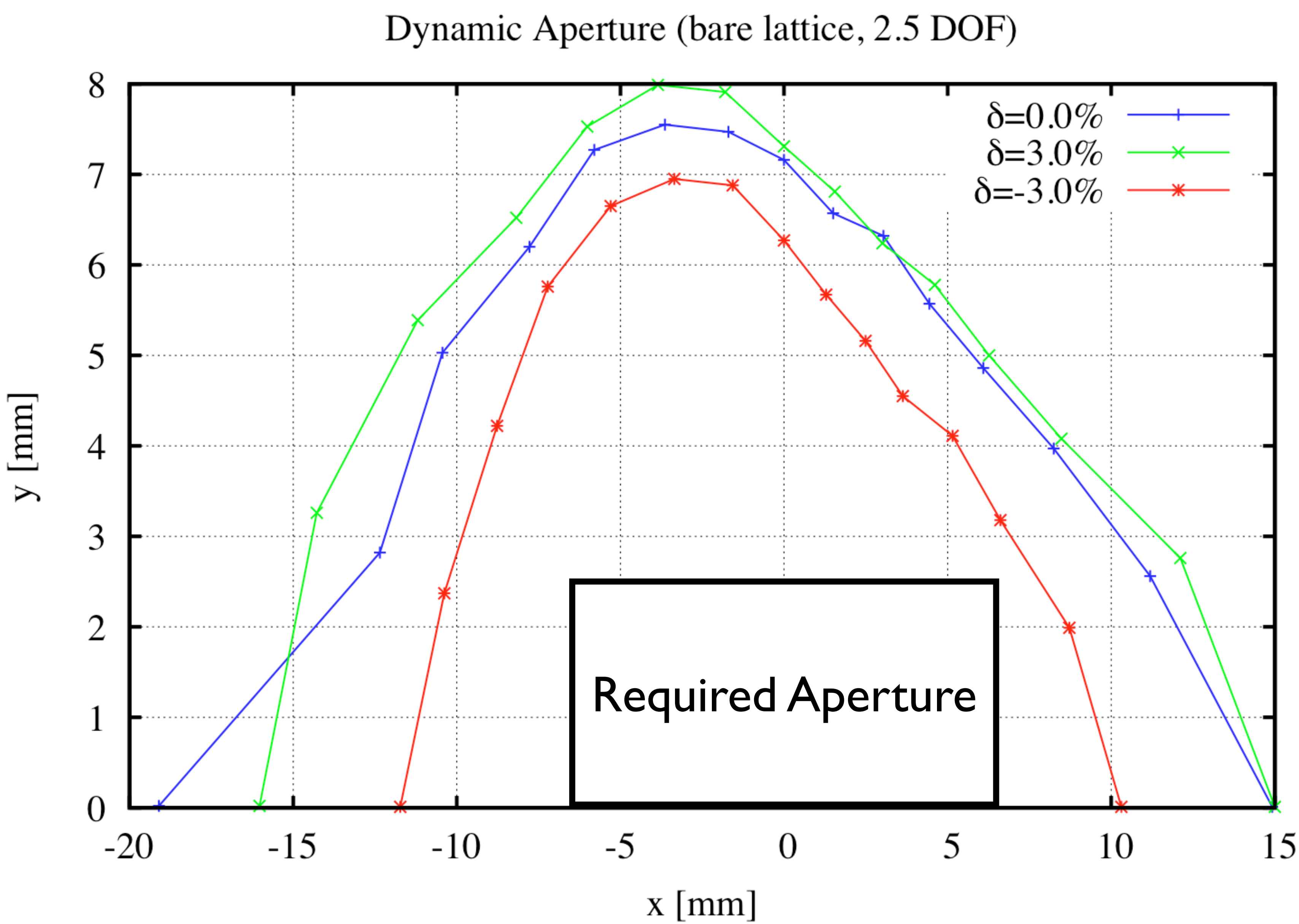
- L = 10 cm
- m = -1283 T/m², -1182 T/m²

SFm / SFo / SFi

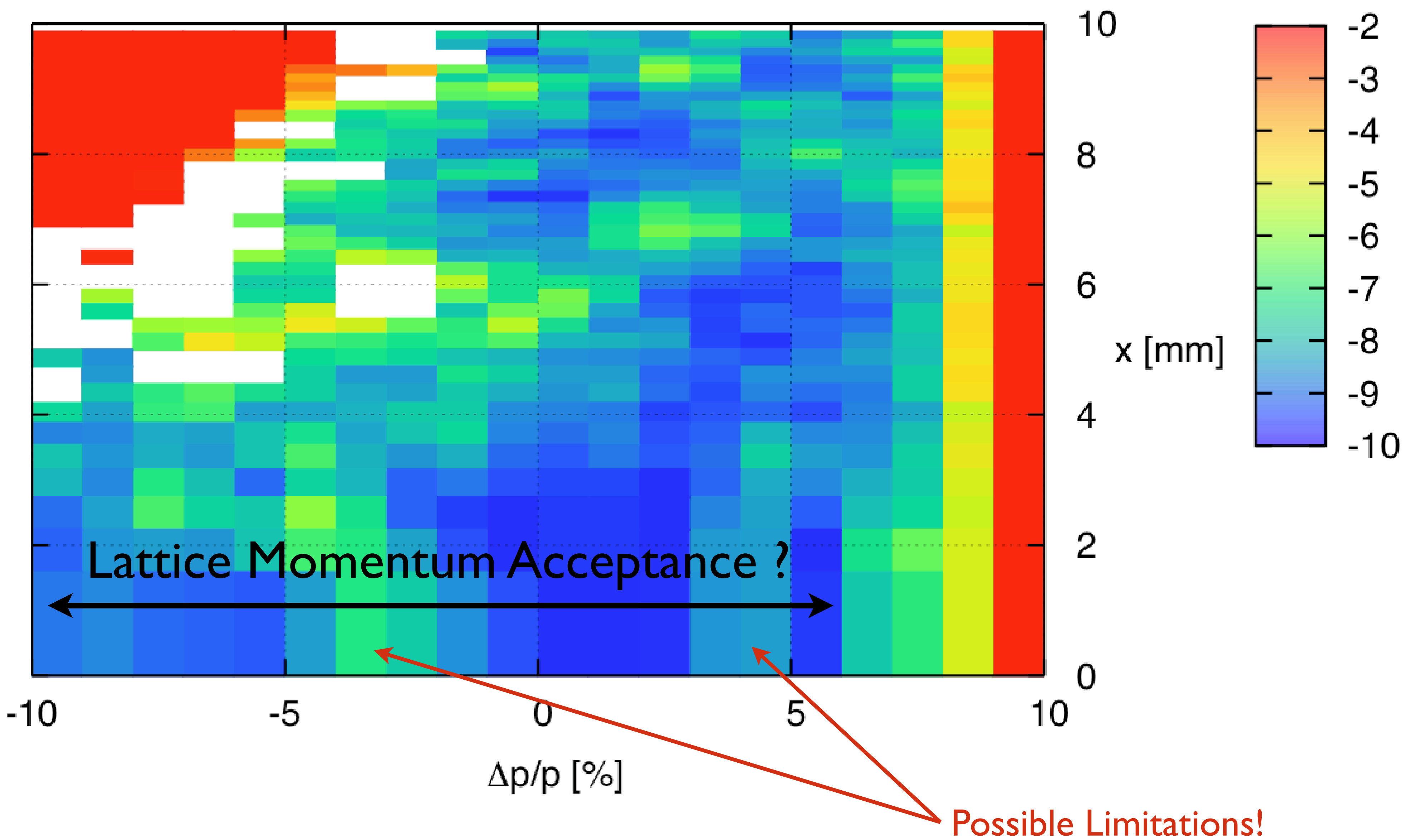
- L = 10 cm
- m = 1926 T/m², 1341 T/m², 2130 T/m²



Lattice Performance (1)



Lattice Performance (2)

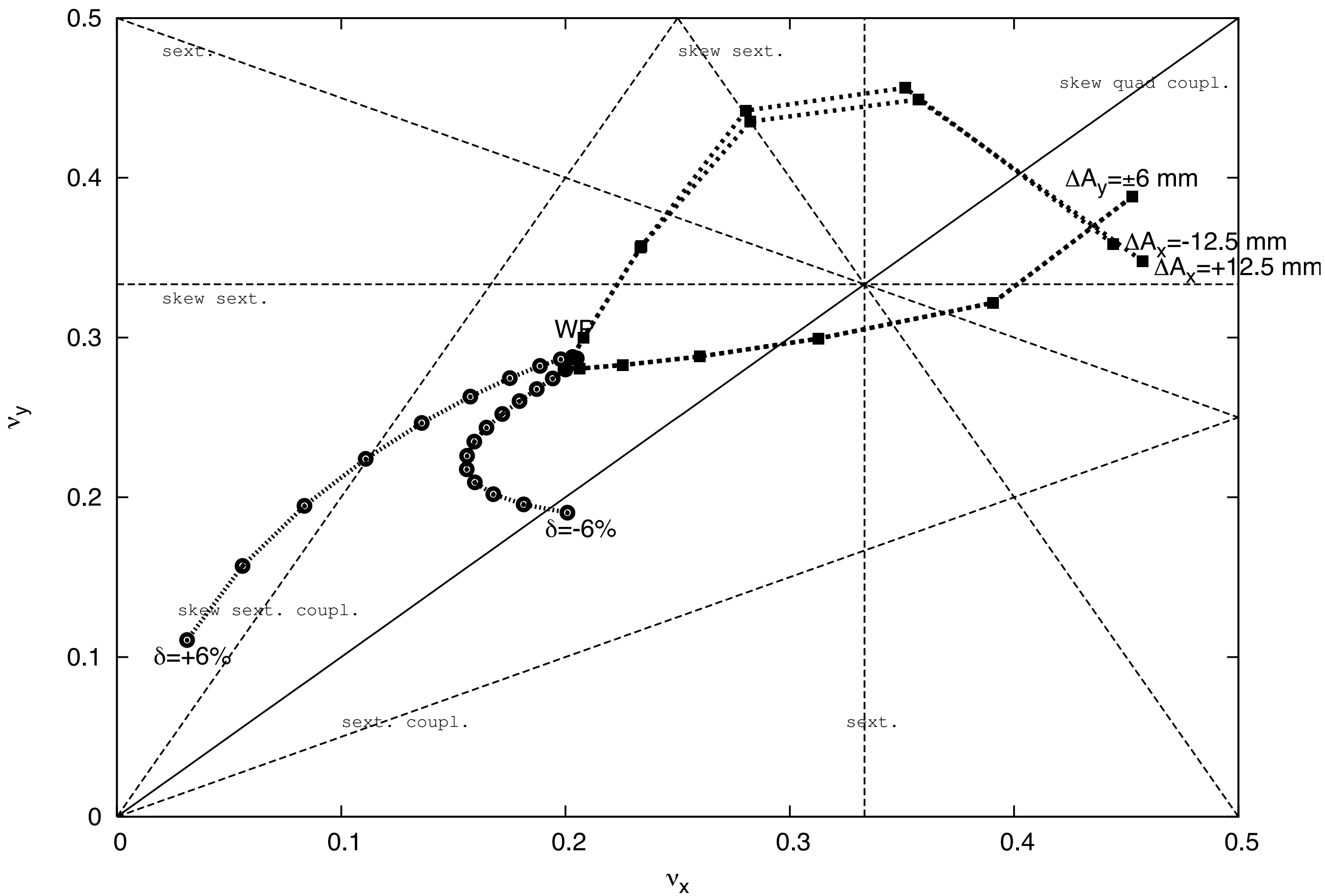


Improving the 3 GeV Lattice with Octupoles

(in collaboration with Andreas Streun, SLS/PSI)

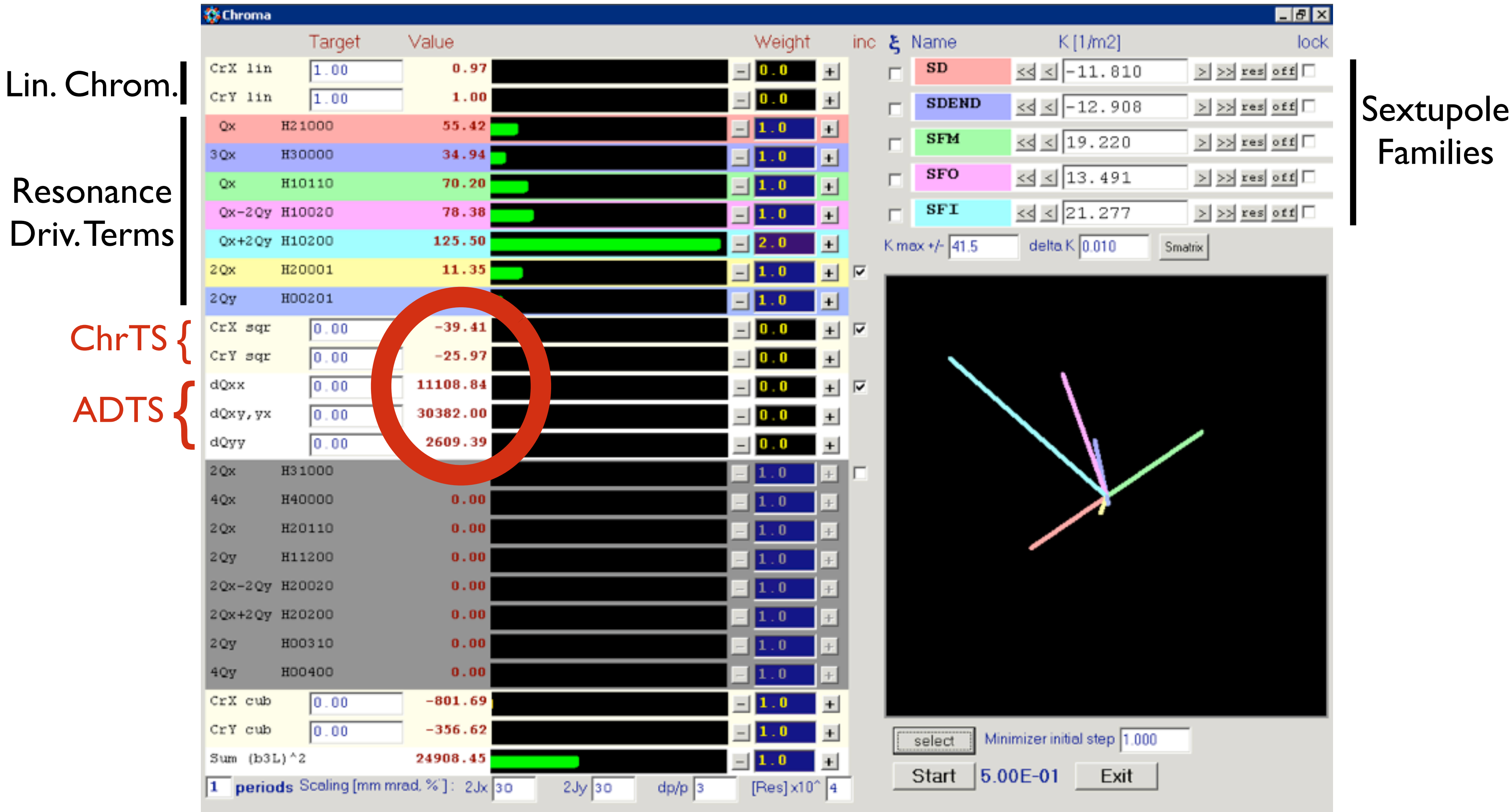
The Problem (1)

- MAX IV has ultra-low emittance → need sufficient dynamic aperture (DA) and momentum acceptance (MA) for efficient capture and sufficient lifetime
- Tune "footprint" is too large → we cross too many resonance lines → bad for DA and MA



The Problem (2)

- Source: Sextupoles (required to correct negative natural chromaticity) drive higher-order terms → strong growth of tune shift with energy (ChrTS) and with amplitude (ADTS)



Octupole Hamiltonian & Driving Terms (1)

- Octupole Hamiltonian

$$H_4 = \frac{b_4}{4}\left[x^4 - 6x^2y^2 + y^4\right]\left(1 - \delta + \delta^2 + \mathcal{O}(\delta^3)\right)$$

b_4 : int. oct. strength in $[\mathrm{T/m^3}]$

$\delta = \Delta p/p$

- Betatron motion on dispersive orbits

$$x = x_\beta + \eta\delta$$

$$\begin{aligned}\longrightarrow H_4 \quad = \quad & \frac{b_4}{4}[(x_\beta^4 - 6x_\beta^2 y^2 + y^4) \\ & +\delta(-x_\beta^4 + 6x_\beta^2 y^2 - y^4 + 4x_\beta^3\eta - 12x_\beta y^2\eta) \\ & +\delta^2(x_\beta^4 - 6x_\beta^2 y^2 + y^4 - 4x_\beta^3\eta + 12x_\beta y^2\eta + 6x_\beta^2\eta^2 - 6y^2\eta^2) \\ & +\mathcal{O}(\delta^3)]\end{aligned}$$

- Resonance basis (Johan Bengtsson)

$$h_{x,y^\pm} = \sqrt{2J_{x,y}}\sqrt{\beta_{x,y}} \times e^{\pm i\varphi_{x,y}} \quad \longrightarrow \quad x_\beta = \frac{1}{2}\left(h_{x^+} + h_{x^-}\right)$$

Octupole Hamiltonian & Driving Terms (2)

- Express octupole Hamiltonian in resonance basis

$$H_4 = \frac{b_4}{4}\frac{1}{16}(h_{x+}^4 + 4h_{x+}^3 h_{x-} + 6h_{x+}^2 h_{x-}^2 + 4h_{x+} h_{x-}^3 + h_{x-}^4)$$
$$+ \ldots \quad \text{in total 83 terms to 2nd order in } \delta$$

- Gather octupole terms by phase

$$\begin{aligned} H_4 &= \sum h_{jklmn} \\ &= \sum \delta^n h_{x+}^j h_{x-}^k h_{y+}^l h_{y-}^m \\ &= \sum \delta^n \left(2J_x\beta_x\right)^{\frac{j+k}{2}} \left(2J_y\beta_y\right)^{\frac{l+m}{2}} e^{i[(j-k)\varphi_x+(l-m)\varphi_y]}. \end{aligned}$$

- Inspect first-order octupole terms

$$(j+k+l+m+n) = 4$$

Octupole Hamiltonian & Driving Terms (3)

- First order octupole terms with phases → drive resonances

$$h_{40000} = \frac{b_4}{64}\beta_x^2(2J_x)^2 e^{i4\varphi_x} \longrightarrow \text{drives } 4Q_x$$

$$h_{31000} = \frac{b_4}{16}\beta_x^2(2J_x)^2 e^{i2\varphi_x} \longrightarrow \text{drives } 2Q_x$$

$$h_{13000} = h_{31000}^* = \frac{b_4}{16}\beta_x^2(2J_x)^2 e^{-i2\varphi_x} \longrightarrow \text{drives } 2Q_x$$

$$h_{04000} = h_{40000}^* = \frac{b_4}{64}\beta_x^2(2J_x)^2 e^{-i4\varphi_x} \longrightarrow \text{drives } 4Q_x$$

$$h_{20200} = -\frac{3b_4}{32}\beta_x\beta_y(2J_x)(2J_y)e^{i(2\varphi_x+2\varphi_y)} \longrightarrow \text{drives } 2Q_x+2Q_y$$

$$h_{20110} = -\frac{3b_4}{16}\beta_x\beta_y(2J_x)(2J_y)e^{i2\varphi_x} \longrightarrow \text{drives } 2Q_x$$

$$h_{20020} = -\frac{3b_4}{32}\beta_x\beta_y(2J_x)(2J_y)e^{i(2\varphi_x-2\varphi_y)} \longrightarrow \text{drives } 2Q_x-2Q_y$$

$$h_{11200} = -\frac{3b_4}{16}\beta_x\beta_y(2J_x)(2J_y)e^{i2\varphi_y} \longrightarrow \text{drives } 2Q_y$$

$$h_{11020} = h_{11200}^* = -\frac{3b_4}{16}\beta_x\beta_y(2J_x)(2J_y)e^{-i2\varphi_y} \longrightarrow \text{drives } 2Q_y$$

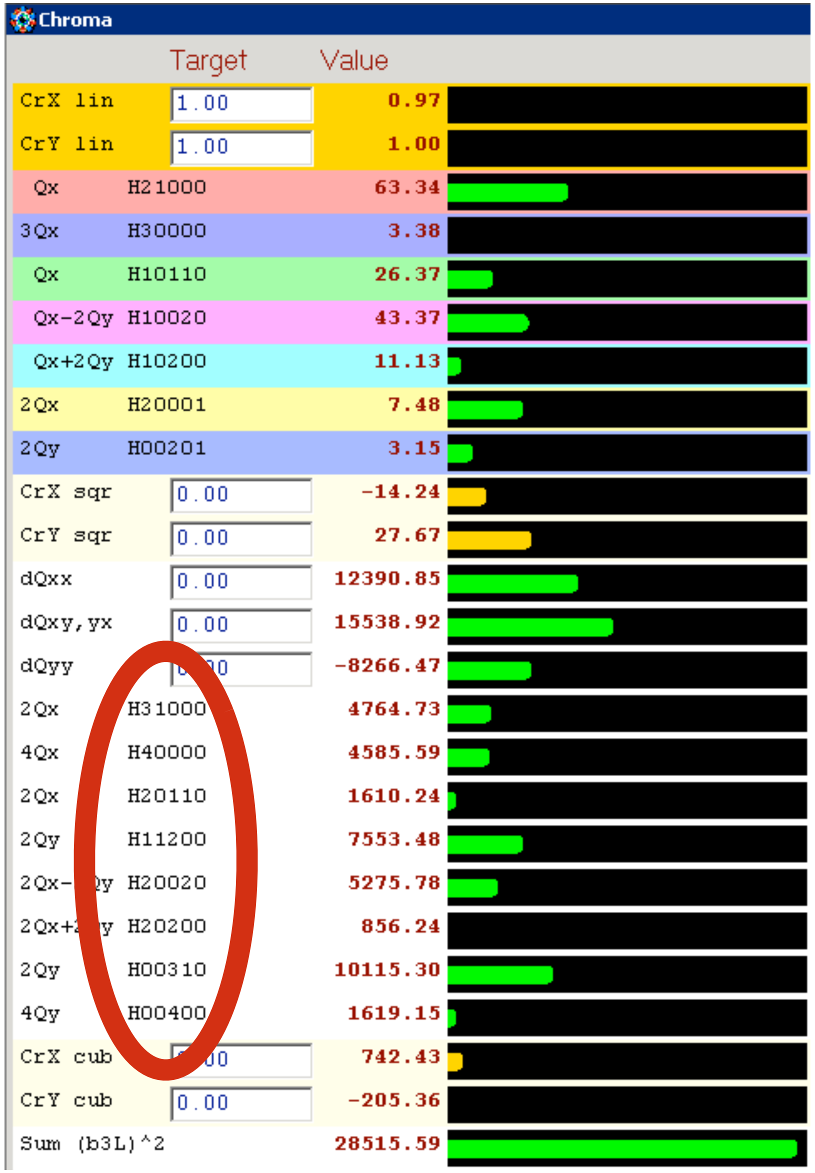
$$h_{02200} = h_{20020}^* = -\frac{3b_4}{32}\beta_x\beta_y(2J_x)(2J_y)e^{-i(2\varphi_x-2\varphi_y)} \longrightarrow \text{drives } 2Q_x-2Q_y$$

$$h_{02110} = h_{20110}^* = -\frac{3b_4}{16}\beta_x\beta_y(2J_x)(2J_y)e^{-i2\varphi_x} \longrightarrow \text{drives } 2Q_x$$

$$h_{02020} = h_{20200}^* = -\frac{3b_4}{32}\beta_x\beta_y(2J_x)(2J_y)e^{i(2\varphi_x+2\varphi_y)} \longrightarrow \text{drives } 2Q_x+2Q_y$$

$$h_{00400} = \frac{b_4}{64}\beta_y^2(2J_y)^2 e^{i4\varphi_y} \longrightarrow \text{drives } 4Q_y$$

... and many more (30 in total)



Octupole Hamiltonian & Driving Terms (4)

- Some first order octupole terms carry no phase → **tune shifts**

$$
\begin{aligned}
h_{22000} &= \frac{3}{8} b_4 \beta_x^2 J_x^2 \\
h_{11110} &= -\frac{3}{2} b_4 \beta_x \beta_y 2 J_x J_y \\
h_{00220} &= \frac{3}{8} b_4 \beta_y^2 J_y^2 \\
h_{11002} &= \frac{6}{4} b_4 \eta^2 \delta^2 \beta_x J_x \\
h_{00112} &= -\frac{6}{4} b_4 \eta^2 \delta^2 \beta_y J_y
\end{aligned}
$$

- Recall that $2\pi\nu$ and J are conjugate action-angle variables

$$
\nu_{x,y} = \frac{1}{2\pi} \frac{\partial H}{\partial J_{x,y}}
$$

Octupole Hamiltonian & Driving Terms (5)

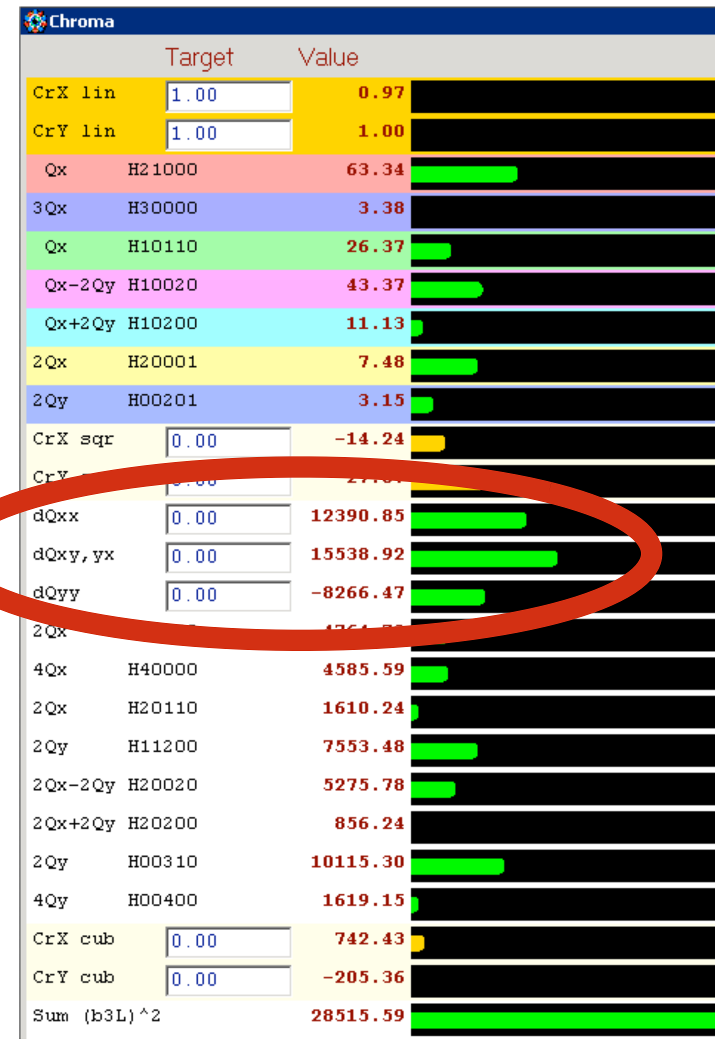
$$\nu_{x,y} = \frac{1}{2\pi}\frac{\partial H}{\partial J_{x,y}} \quad \longrightarrow \quad \frac{\partial \Delta\nu_{x,y}}{\partial J_{x,y}} = \frac{1}{2\pi}\frac{\partial^2 H}{\partial J_{x,y}^2}$$

- 3 driving terms for ADTS

$$\Rightarrow \frac{\partial \Delta\nu_x}{\partial J_x} = \boxed{\frac{3}{8\pi} b_4 \beta_x^2}$$

$$\Rightarrow \frac{\partial \Delta\nu_y}{\partial J_y} = \boxed{\frac{3}{8\pi} b_4 \beta_y^2}$$

$$\Rightarrow \frac{\partial \Delta\nu_x}{\partial J_y} = \frac{\partial \Delta\nu_y}{\partial J_x} = \boxed{-\frac{3}{4\pi} b_4 \beta_x \beta_y}$$



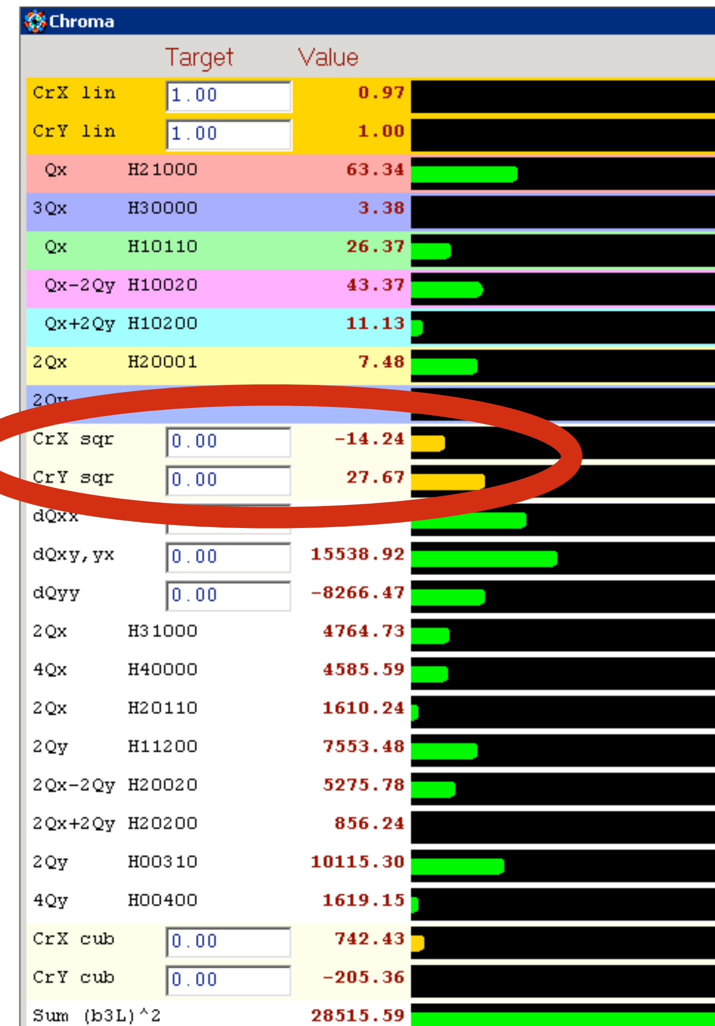
Octupole Hamiltonian & Driving Terms (6)

$$\nu_{x,y} = \frac{1}{2\pi}\frac{\partial H}{\partial J_{x,y}} \quad \longrightarrow \quad \frac{\partial^2 \Delta\nu_{x,y}}{\partial\delta^2} = \frac{1}{2\pi}\frac{\partial^2}{\partial\delta^2}\frac{\partial H}{\partial J_{x,y}}$$

- 2 driving terms for ChrTS (quadratic chromaticity)

$$\xi^{(2)}_{x,y} = \frac{1}{2}\frac{\partial^2 \Delta\nu_{x,y}}{\partial\delta^2} = \frac{1}{4\pi}\frac{\partial^2}{\partial\delta^2}\frac{\partial H}{\partial J_{x,y}}$$

$$\Rightarrow \xi^{(2)}_{x,y} = \boxed{\pm\frac{3}{4\pi}b_4\eta^2\beta_{x,y}}$$



How Octupoles Can be Used to Solve Problems (1)

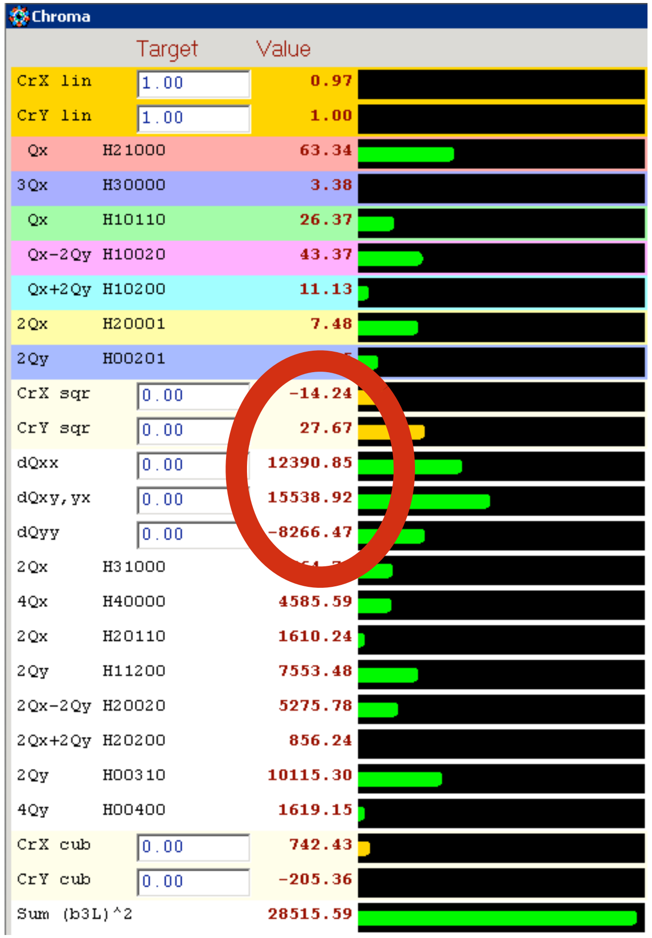
- Matrix formalism for 5 higher-order driving terms and n octupole families

$$\begin{pmatrix} \partial\nu_x/\partial J_x \\ \partial\nu_x/\partial J_y \\ \partial\nu_y/\partial J_y \\ \partial^2\nu_x/\partial\delta^2 \\ \partial^2\nu_y/\partial\delta^2 \end{pmatrix} = \mathcal{M}_{5\times n} \begin{pmatrix} (b_4 l)_1 \\ \vdots \\ (b_4 l)_n \end{pmatrix}$$

5 known expressions evaluated at n octupole locations (β_x, β_y, η)

- From OPA we know what the 5 higher-order driving terms generated by the sextupole are...

Reverse the problem: what octupole strengths $(b_4 l)_j$ are required to exactly cancel these 5 terms?



How Octupoles Can be Used to Solve Problems (2)

- Inversion or SVD → use octupoles to cancel higher-order terms generated by sextupoles

$$\Longrightarrow \mathcal{M}^{-1}_{5\times n} \times (-1) \times \begin{pmatrix} \partial\nu_x/\partial J_x \\ \vdots \\ \partial^2\nu_y/\partial\delta^2 \end{pmatrix} = \begin{pmatrix} (b_4 l)_1 \\ \vdots \\ (b_4 l)_n \end{pmatrix}$$

SVD — cancellation

5 higher-order driving terms generated by sextupoles

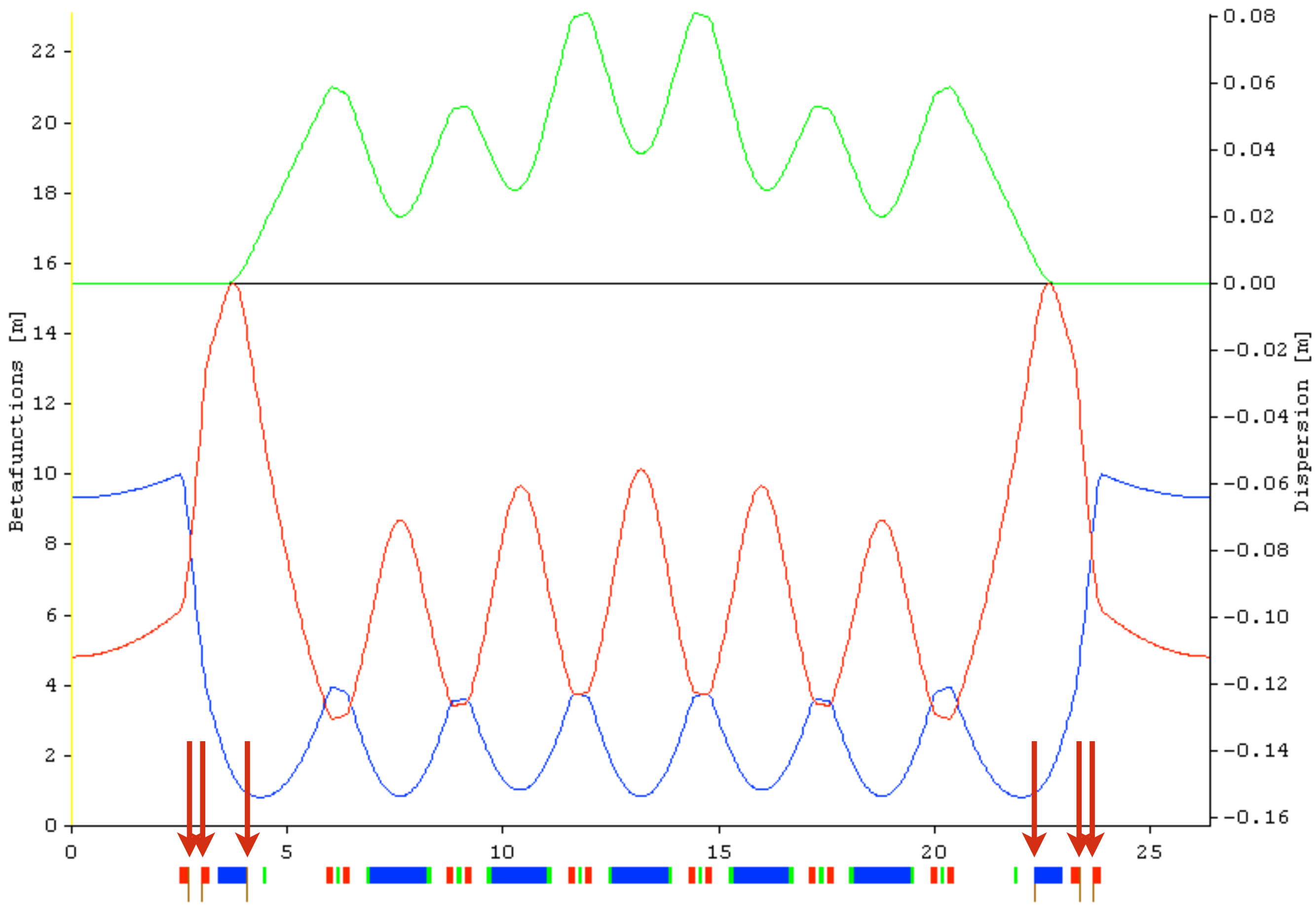
n resulting oct. strengths

- If n = 5 → matrix inversion → perfect cancellation
- If n > 5 → SVD required → perfect cancellation, minimize req. oct. strength
- If n < 5 → SVD required → minimize driving terms

- If no chromatic octupoles available → reduce to 3-dim. harmonic problem

Octupoles in the MAX IV Lattice (1)

- Studied different layouts with up to six octupole families
- Chromatic octupoles appear to be detrimental (higher-order chromatic terms)
- Harmonic octupoles at right locations are very efficient (5cm length!)



Periods		20
Length	[m]	527.760
TuneX		42.20000
TuneY		14.28000
ChromX		-49.528
ChromY		-42.659
Alpha	[xE-3]	0.310
Jx		1.84030
Energy	[GeV]	3.000
EmitXo	[nm rd]	0.336
dE/turn	[keV]	357.0
Espread	[xE-3]	0.772
TauX	[ms]	16.079
TauY	[ms]	29.591
TauE	[ms]	25.516
Location		END
Position	m	26.388
BetaX	m	9.330
AlphaX		0.0000
BetaY	m	4.816
AlphaY		0.0000
Disp.	m	0.0000
dD/ds	rad	0.0000
PhiX/2pi		2.1100
PhiY/2pi		0.7140
curly H	m	0.000000

Octupoles in the MAX IV Lattice (2)

- Three weak and compact families are sufficient for near-perfect cancellation of ADTS while retaining sufficiently small ChrTS

Chroma

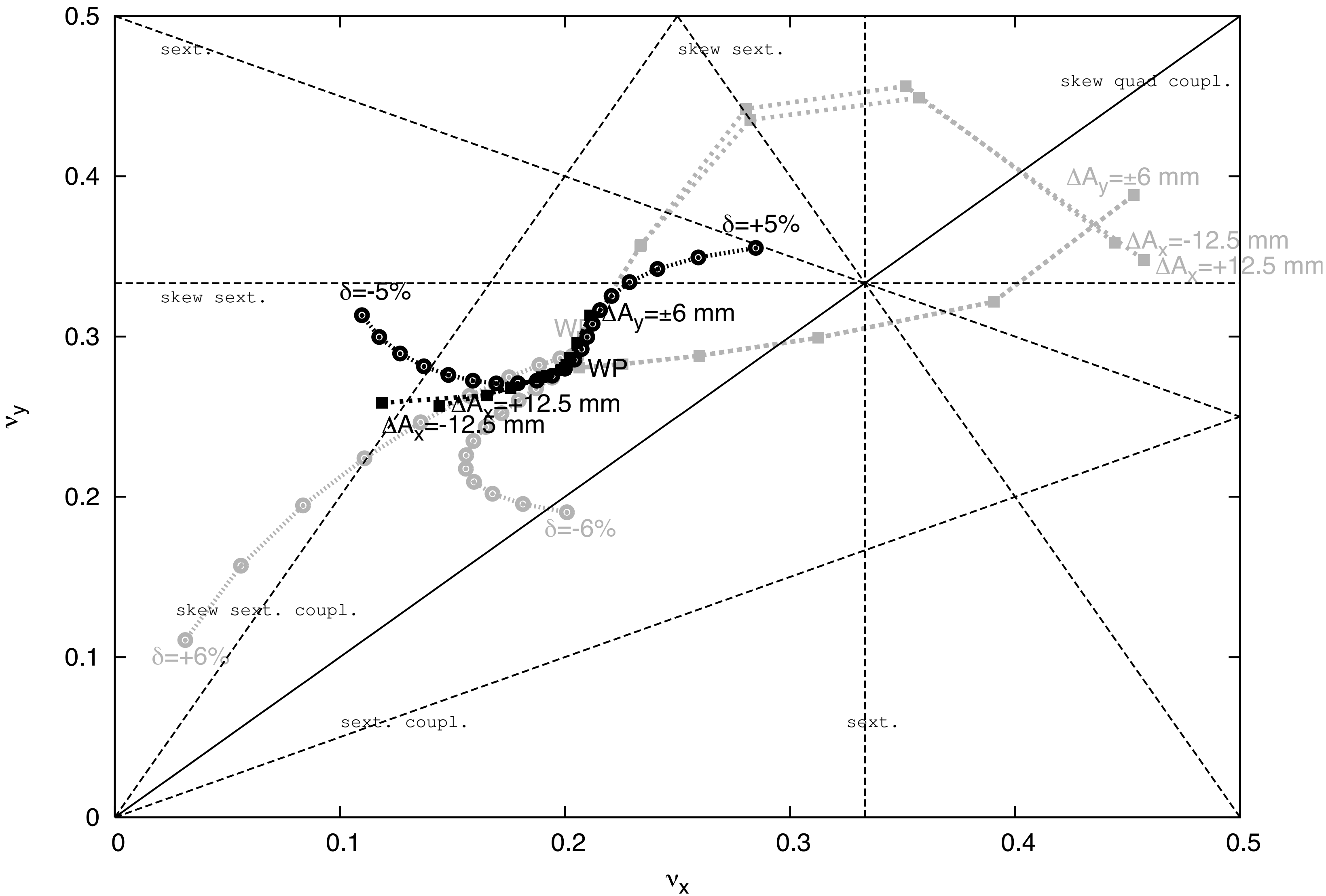
		Target	Value
CrX lin		1.00	0.97
CrY lin		1.00	1.00
Qx	H21000		55.42
3Qx	H30000		34.94
Qx	H10110		70.20
Qx-2Qy	H10020		78.38
Qx+2Qy	H10200		125.50
2Qx	H20001		11.35
2Qy	H00201		5.37
CrX sqr		0.00	-39.41
CrY sqr		0.00	-25.97
dQxx		0.00	11108.84
dQxy,yx		0.00	30382.00
dQyy		0.00	2609.39

Chroma

		Target	Value
CrX lin		0.00	0.97
CrY lin		0.00	1.00
Qx	H21000		31.74
3Qx	H30000		1.18
Qx	H10110		12.17
Qx-2Qy	H10020		22.20
Qx+2Qy	H10200		35.20
2Qx	H20001		7.93
2Qy	H00201		1.72
CrX sqr		0.00	-43.16
CrY sqr		0.00	58.10
dQxx		0.00	-2.23
dQxy,yx		0.00	3.68
dQyy		0.00	-6.77

Octupoles in the MAX IV Lattice (3)

- Within required DA we cross not a single regular resonance below 4th order
- MA of the lattice better than ±5%



Octupoles in the MAX IV Lattice (4)