Low Emittance Lattice Design with Unconventional Magnets

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Different Approaches to Low Emittance

Natural horizontal equilibrium emittance for a flat storage ring:

$$\varepsilon_0[\mathrm{nm\,rad}] = 1470\,E[\mathrm{GeV}]^2\,\frac{I_5}{J_x I_2}, \quad J_x = 1 - \frac{I_4}{I_2}$$

$$I_2 = \oint \frac{ds}{\rho^2}, \quad I_4 = \oint \frac{\eta}{\rho}(\frac{1}{\rho^2} + 2b_2)ds, \quad I_5 = \oint \frac{\mathcal{H}}{\rho^3}ds$$

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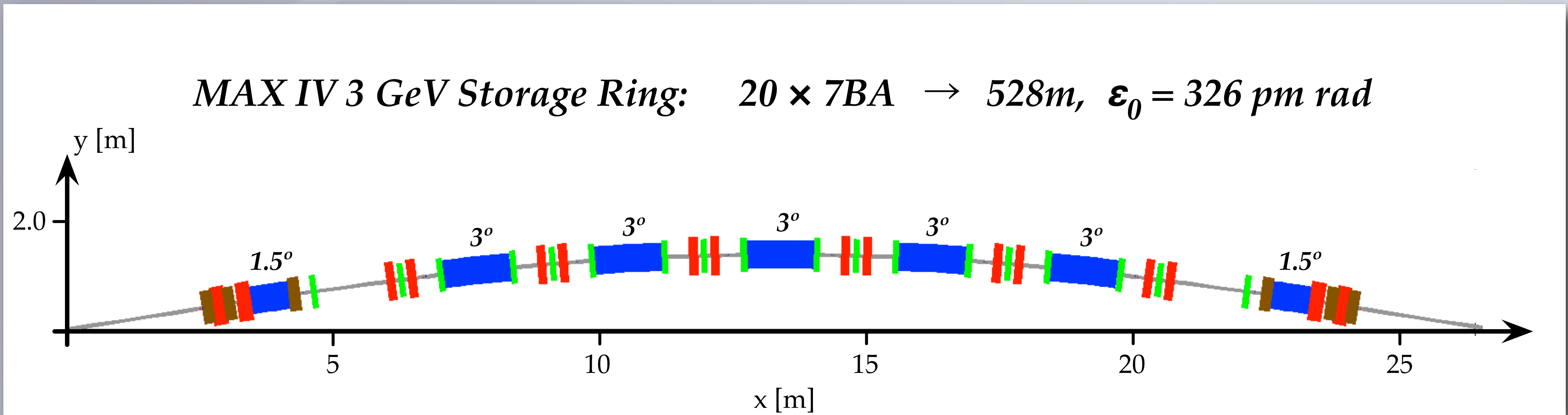
- Dispersive straights, damping wigglers, longitudinal gradient bends, etc.
- Multibend achromat (MBA) → many weak bends → relax optics constraints

$$\varepsilon_0[\mathrm{nm\,rad}] = \frac{7.8}{J_x}\,E[\mathrm{GeV}]^2\,\Phi[^\circ]^3\,\frac{F(\beta_x,\eta)_\rho}{12\sqrt{15}}, \qquad \boxed{\Phi[^\circ]^3 \propto \frac{1}{N_{\mathrm{bend}}^3}}$$

Multibend Achromat Lattices

- Example: MAX IV 3 GeV storage ring → 7-bend achromat, ε_0 = 326 pm rad
- Need very compact optics to prevent MBA ring from becoming large & costly



PRST-AB **12**, 120701 (2009)

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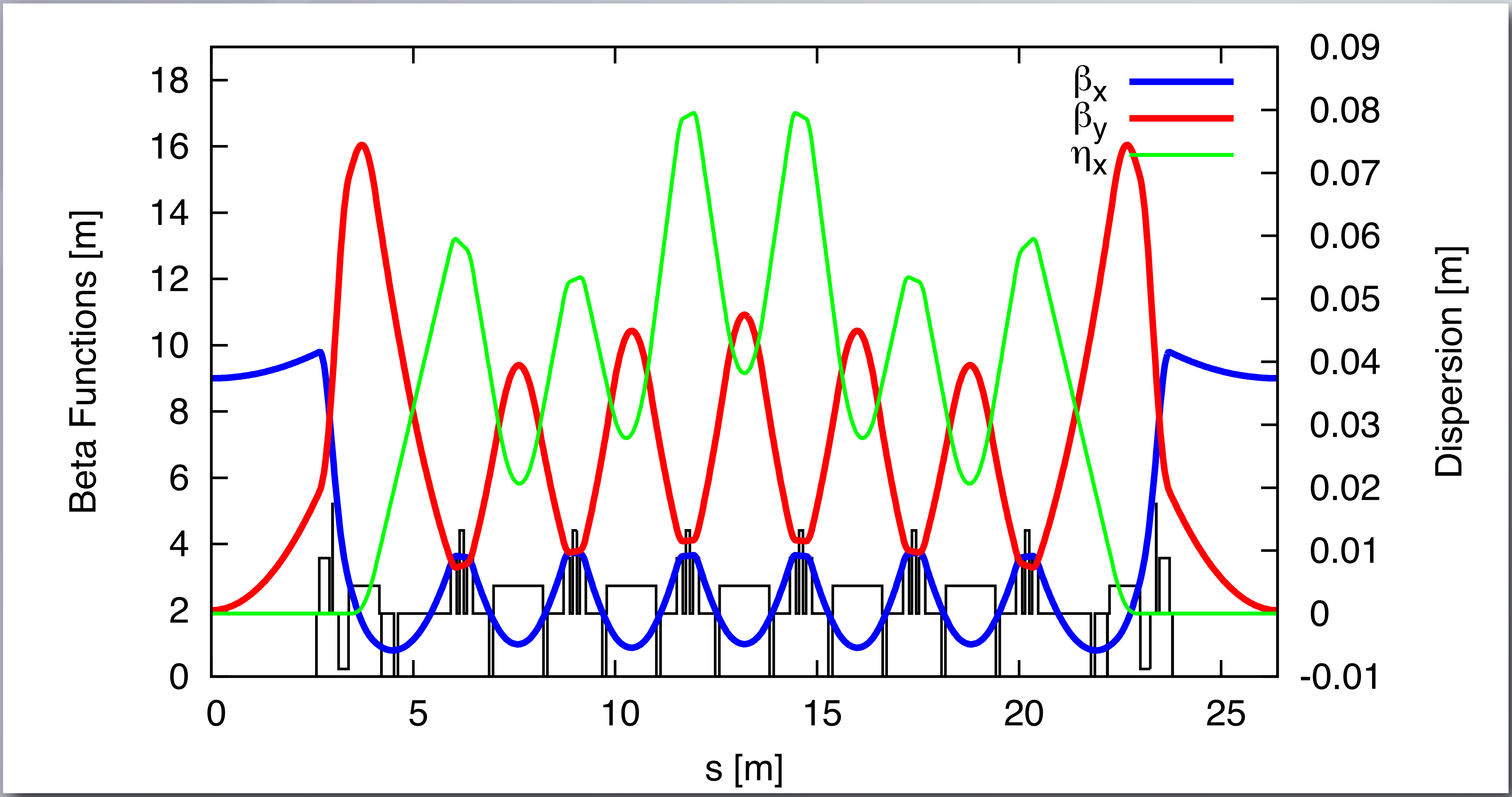

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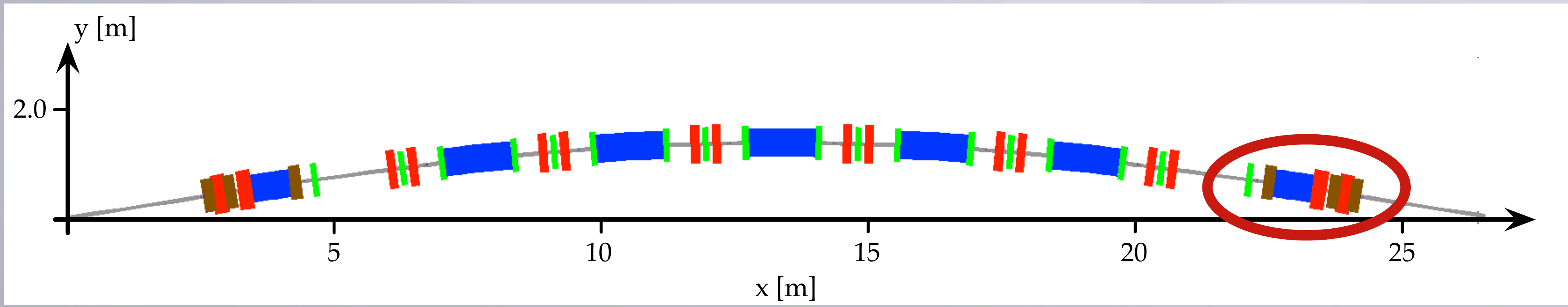
  - ➡ Fully integrated magnet design



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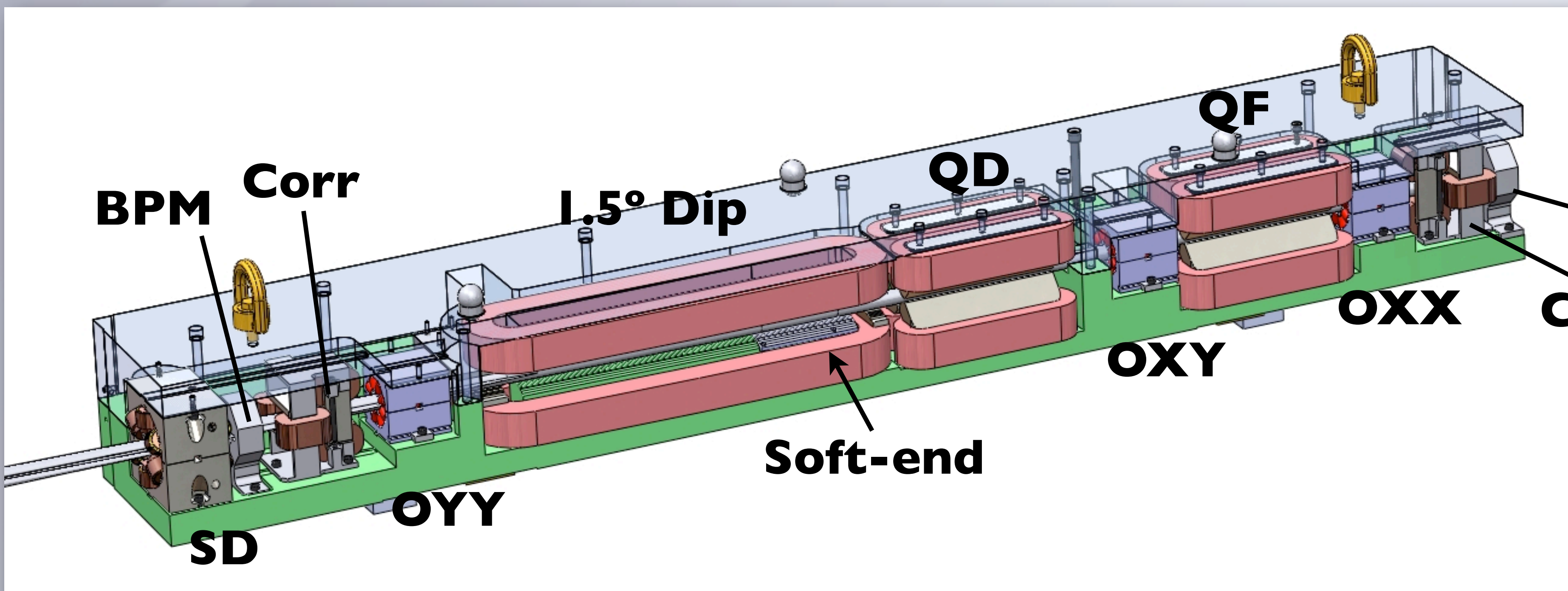

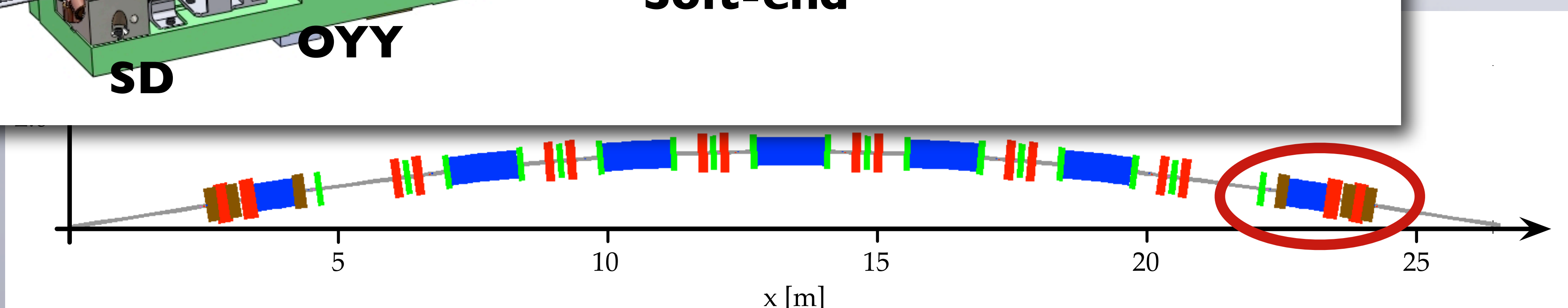

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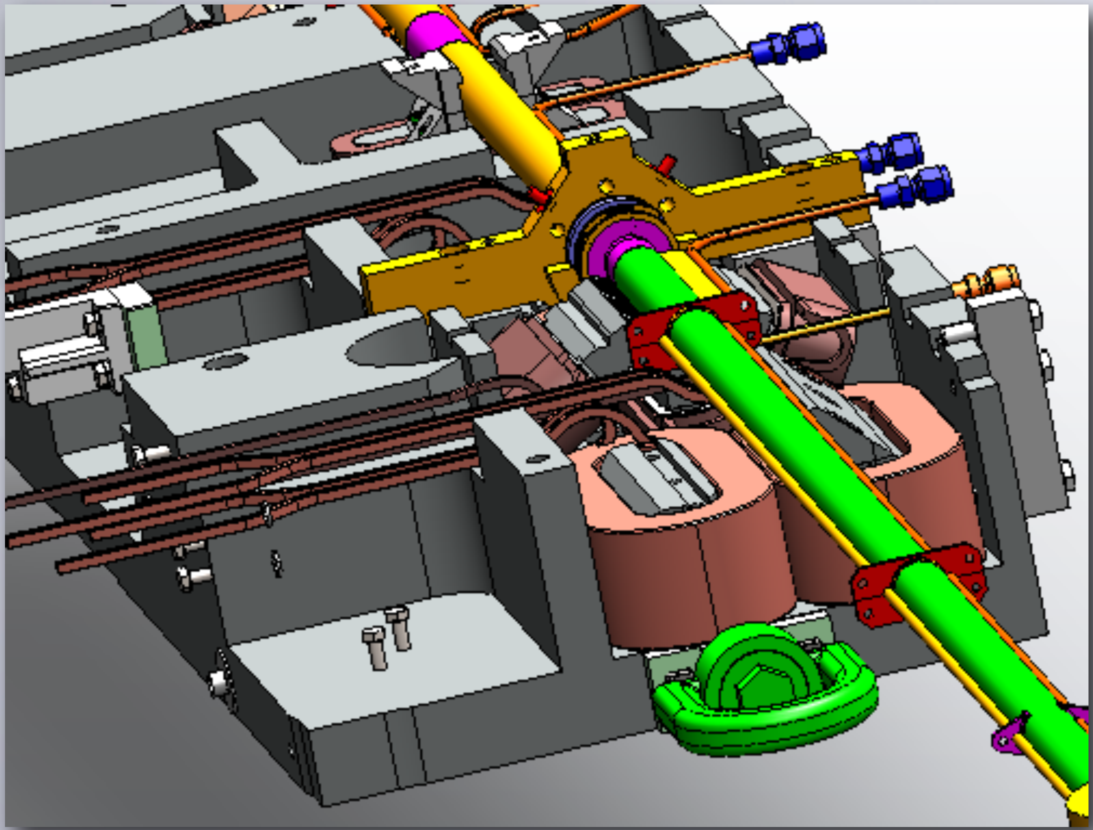
    - ➡ Small magnets with narrow apertures (vacuum issues → NEG coating)

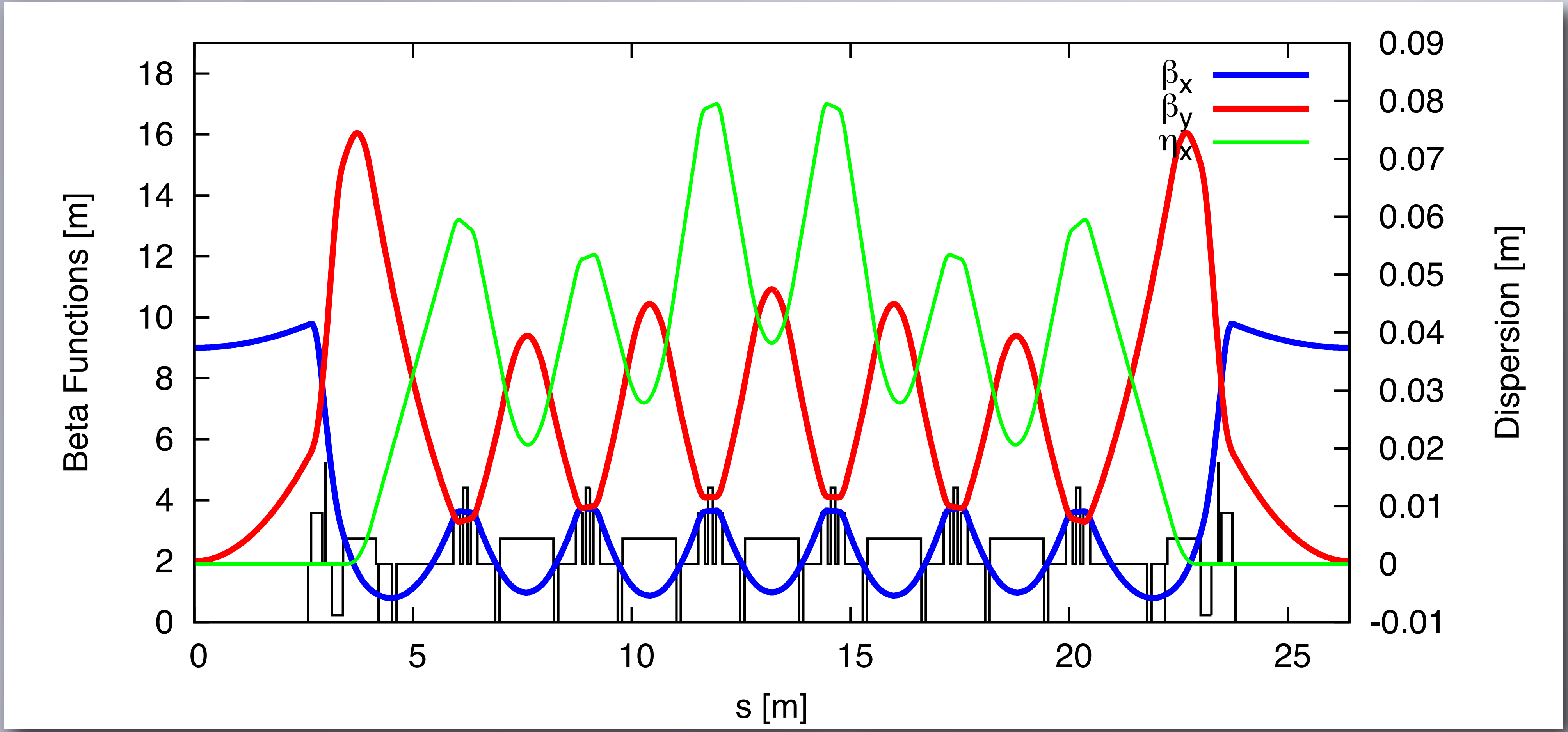


IPAC'11, TUPS016

The Nonlinear Optics Challenge in a MBA Lattice

- Strong focusing optics → large negative natural chromaticity, small dispersion
  - ➡ Require strong sextupoles → adjust two chromatic terms (linear chromaticity)

$$h_{11001} \rightarrow \xi_x^{(1)} \qquad h_{00111} \rightarrow \xi_y^{(1)}$$



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$$h_{11001} \rightarrow \xi_x^{(1)} \qquad h_{00111} \rightarrow \xi_y^{(1)}$$

- But additional 3 chromatic and 5 geometric terms (RDTs) also need to be minimized

$$h_{10002} \rightarrow \frac{d\eta}{d\delta}$$

$$h_{20001} \rightarrow \frac{d\beta_x}{d\delta}$$

$$h_{00201} \rightarrow \frac{d\beta_y}{d\delta}$$

$$h_{21000} \rightarrow \nu_x$$

$$h_{30000} \rightarrow 3\nu_x$$

$$h_{10110} \rightarrow \nu_x$$

$$h_{10200} \rightarrow \nu_x + 2\nu_y$$

$$h_{10020} \rightarrow \nu_x - 2\nu_y$$

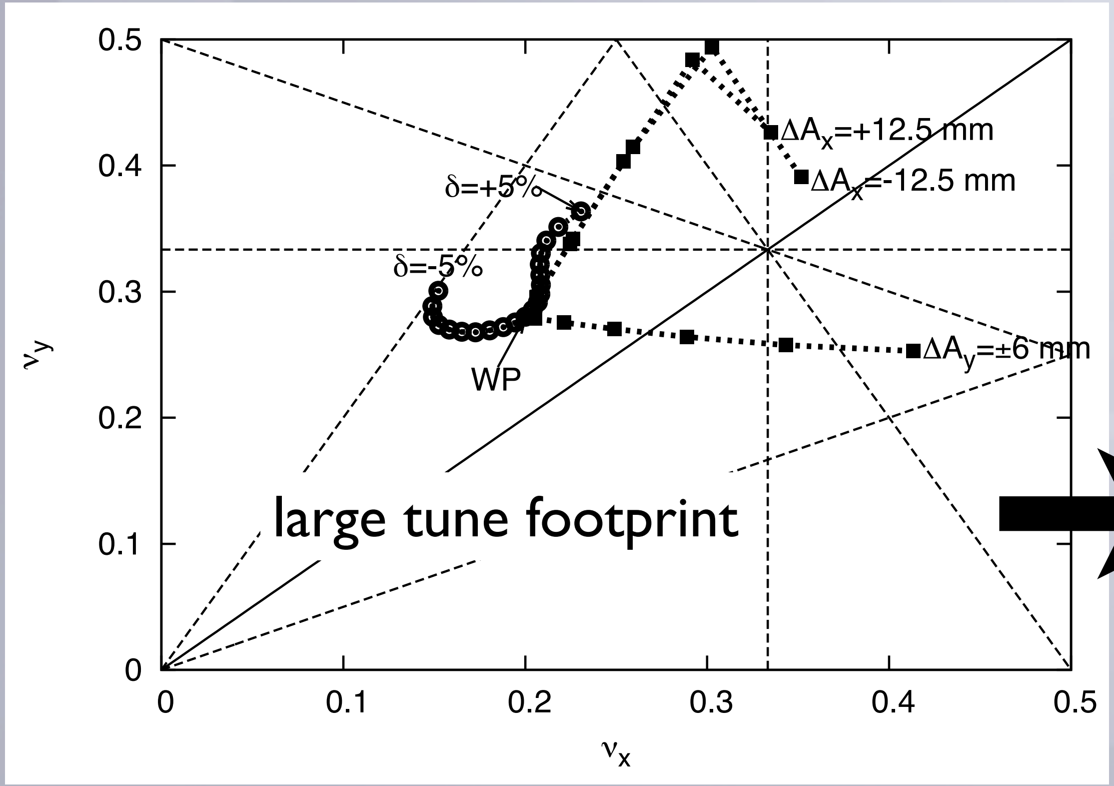
... and this is just to first order!

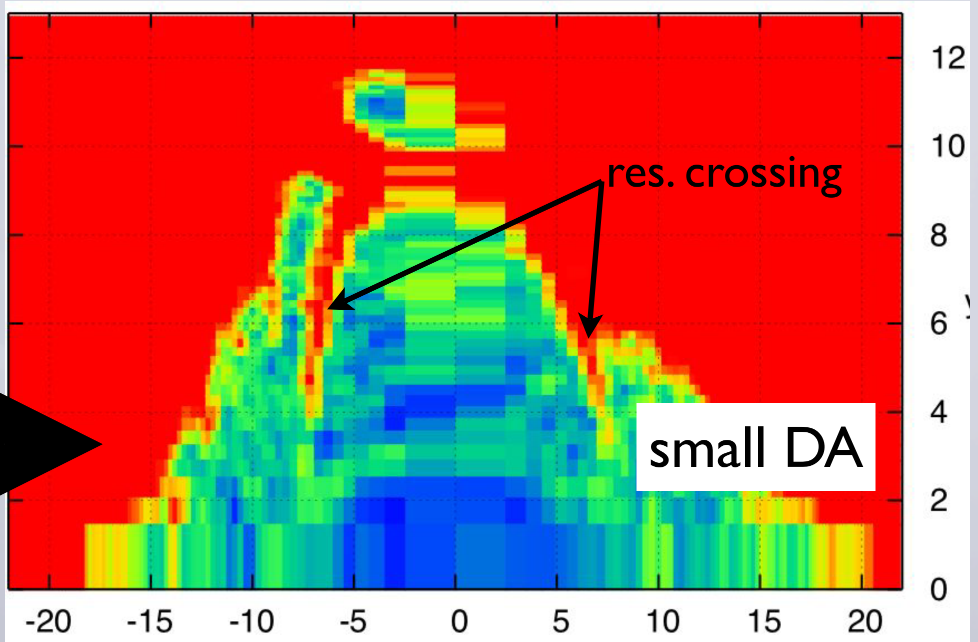
The Nonlinear Optics Challenge in a MBA Lattice

- In addition, second-order effects determine amplitude-dependent tune shift (ADTS) and 2nd-order chromaticity

$$\frac{\partial \nu_x}{\partial J_x}, \frac{\partial \nu_y}{\partial J_y}, \frac{\partial \nu_x}{\partial J_y} = \frac{\partial \nu_y}{\partial J_x}, \qquad \xi_x^{(2)}, \xi_y^{(2)}$$

➡ Control of these terms is crucial to reduce tune footprint → increase dynamic aperture (DA) and momentum acceptance (MA)





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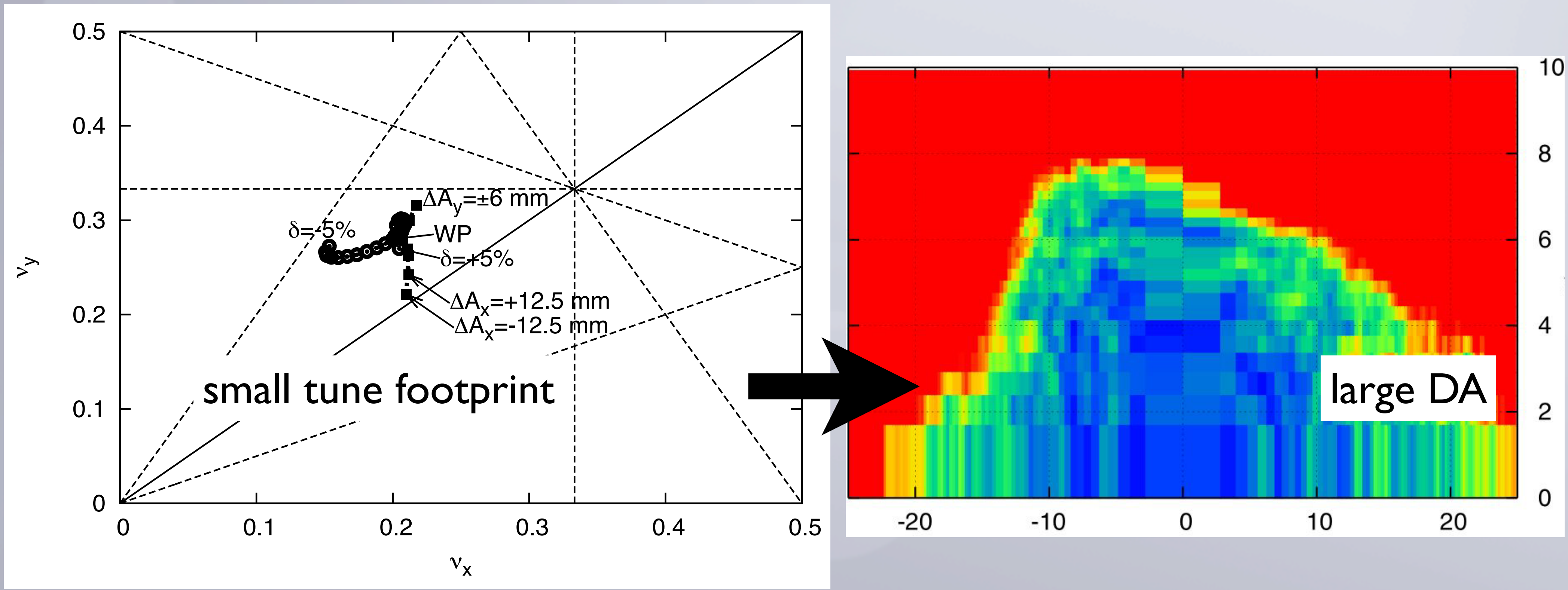

A Strategy to Optimize Nonlinear Dynamics in a MBA

- Weak dispersion → second-order corrections with sextupoles require extra strength → RDTs increase → potential run-away cycle
  - ➡ Instead, make efficient use of multipoles:
    - Use sextupoles to correct linear chromaticity and minimize first-order RDTs
    - Introduce octupoles to correct ADTS and/or 2nd-order chromaticity
    - Introduce decapoles to correct 3rd-order chromaticity
    - ...
  - ➡ Comparably weak octupoles/decapoles are sufficient to compensate higher-order effects from strong sextupoles

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A Strategy to Optimize Nonlinear Dynamics in a MBA

- Example: octupole Hamiltonian

$$\begin{pmatrix} \partial\Delta\nu_x/\partial J_x \\ \partial\Delta\nu_y/\partial J_y \\ \partial\Delta\nu_x/\partial J_y \\ \partial^2\Delta\nu_x/\partial\delta^2 \\ \partial^2\Delta\nu_y/\partial\delta^2 \end{pmatrix} = \frac{3}{8\pi} \begin{pmatrix} (\beta_x)_1^2 & \cdots & (\beta_x)_{N_\mathrm{oct}}^2 \\ -2(\beta_x\beta_y)_1 & \cdots & -2(\beta_x\beta_y)_{N_\mathrm{oct}} \\ (\beta_y)_1^2 & \cdots & (\beta_y)_{N_\mathrm{oct}}^2 \\ 4(\eta^2\beta_x)_1 & \cdots & 4(\eta^2\beta_x)_{N_\mathrm{oct}} \\ -4(\eta^2\beta_y)_1 & \cdots & -4(\eta^2\beta_y)_{N_\mathrm{oct}} \end{pmatrix} \begin{pmatrix} (b_4L)_1 \\ \vdots \\ (b_4L)_{N_\mathrm{oct}} \end{pmatrix}$$

Tune shifts $\vec{\delta\nu}_\mathrm{oct}$ — Optics $\mathbf{B}_\mathrm{oct}$ — Octupole family strengths $\vec{b}_4$

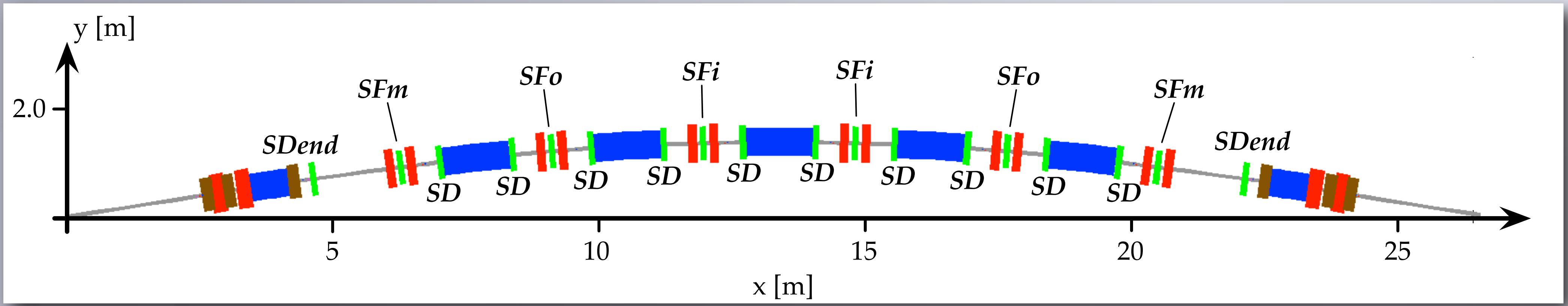
➡ Pseudoinversion (SVD) → octupole strengths required to cancel tune shifts of bare sextupole lattice

$$\vec{b}_4 = \mathbf{B}_\mathrm{oct}^{-1}\,\vec{\delta\nu}_\mathrm{oct} = -\mathbf{B}_\mathrm{oct}^{-1}\,\vec{\delta\nu}$$

- OPA is an extremely useful tool for this work → direct interaction, weighting, display of sextupole kicks and RDTs in complex plane, RDT minimization, optimization of octupoles/decapoles (SVD), etc. http://people.web.psi.ch/streun/opa
- But ultimately, lattice performance determined by magnet errors, misalignments, ID matching, IBS, etc. → require tracking to verify → Tracy-3 for MAX IV SRs

Nonlinear Optimization of the MAX IV 3 GeV SR

- Introduce sextupoles to correct linear chromaticity & minimize first-order RDTs



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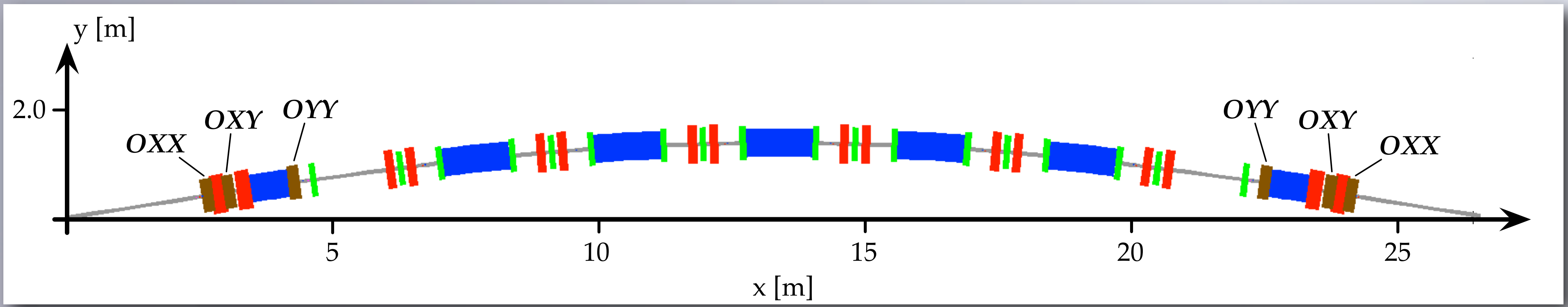
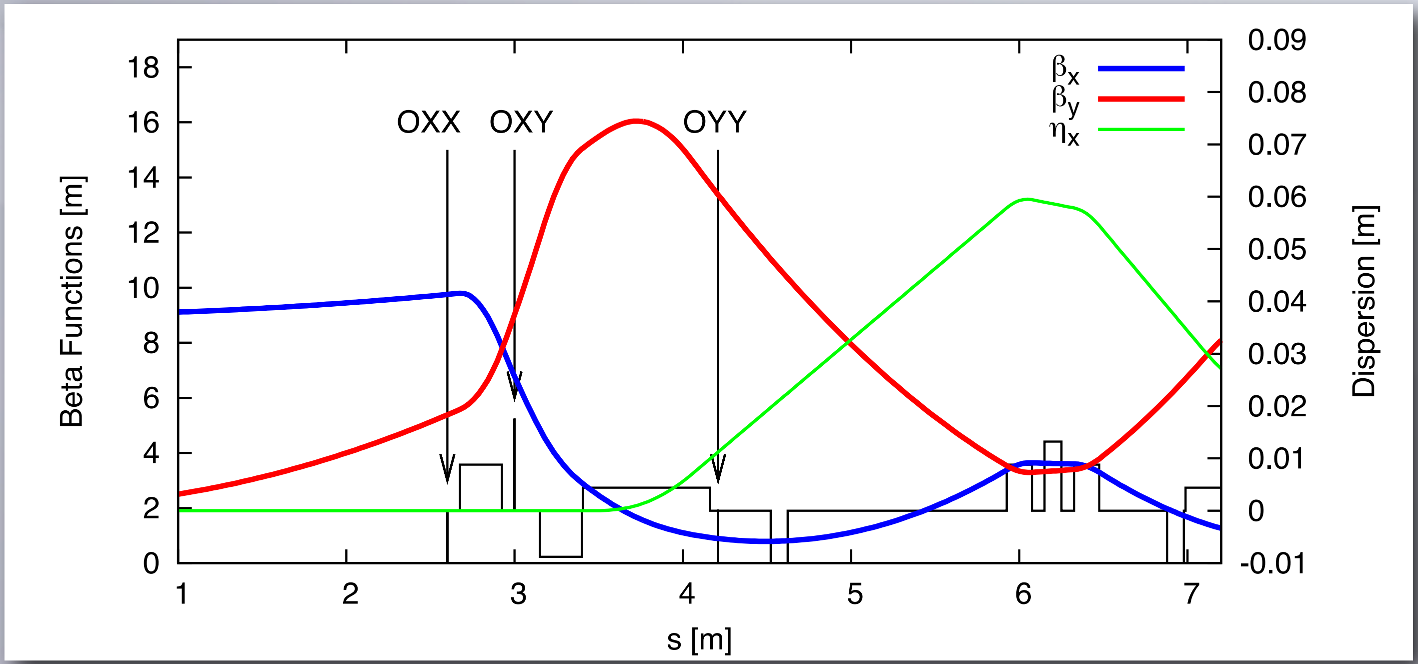
- Introduce three non-dispersive octupole families to adjust three ADTS terms





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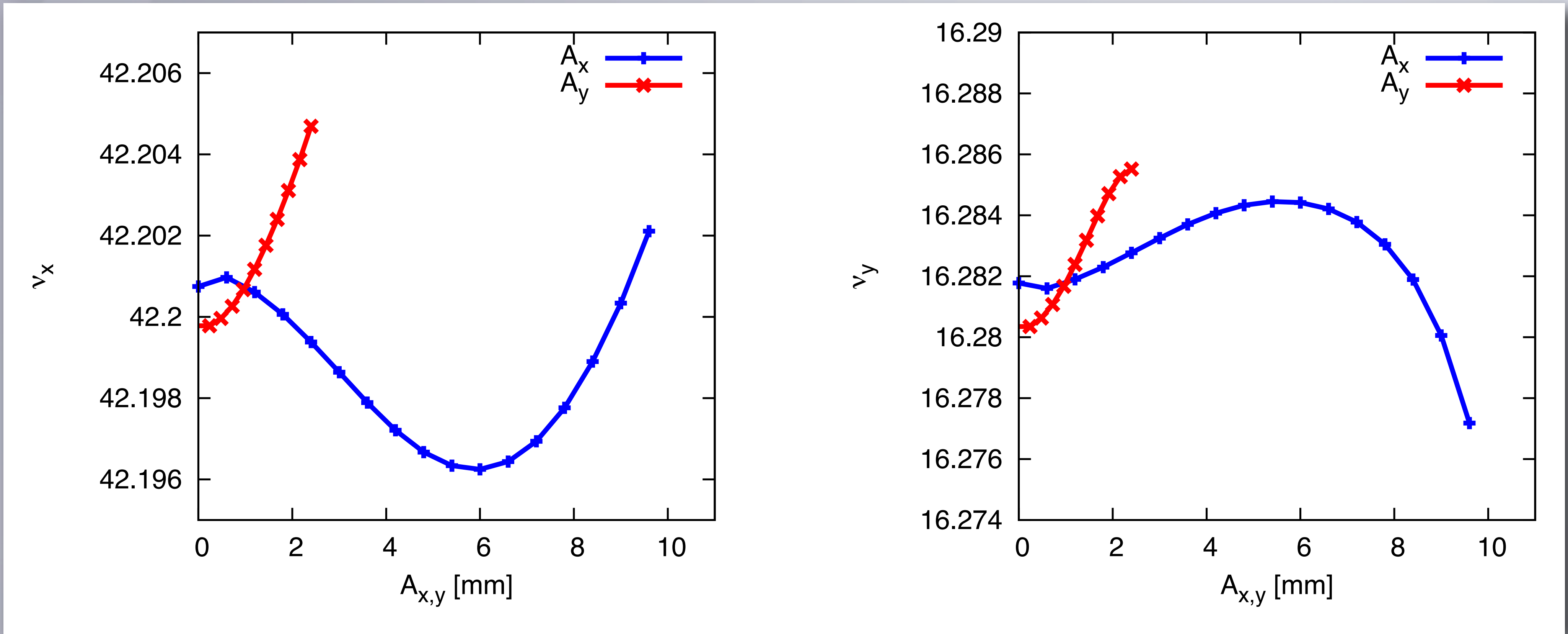
  - ➡ Instead of first-order cancellation, adjust first-order terms to minimize resulting overall ADTS across area of interest



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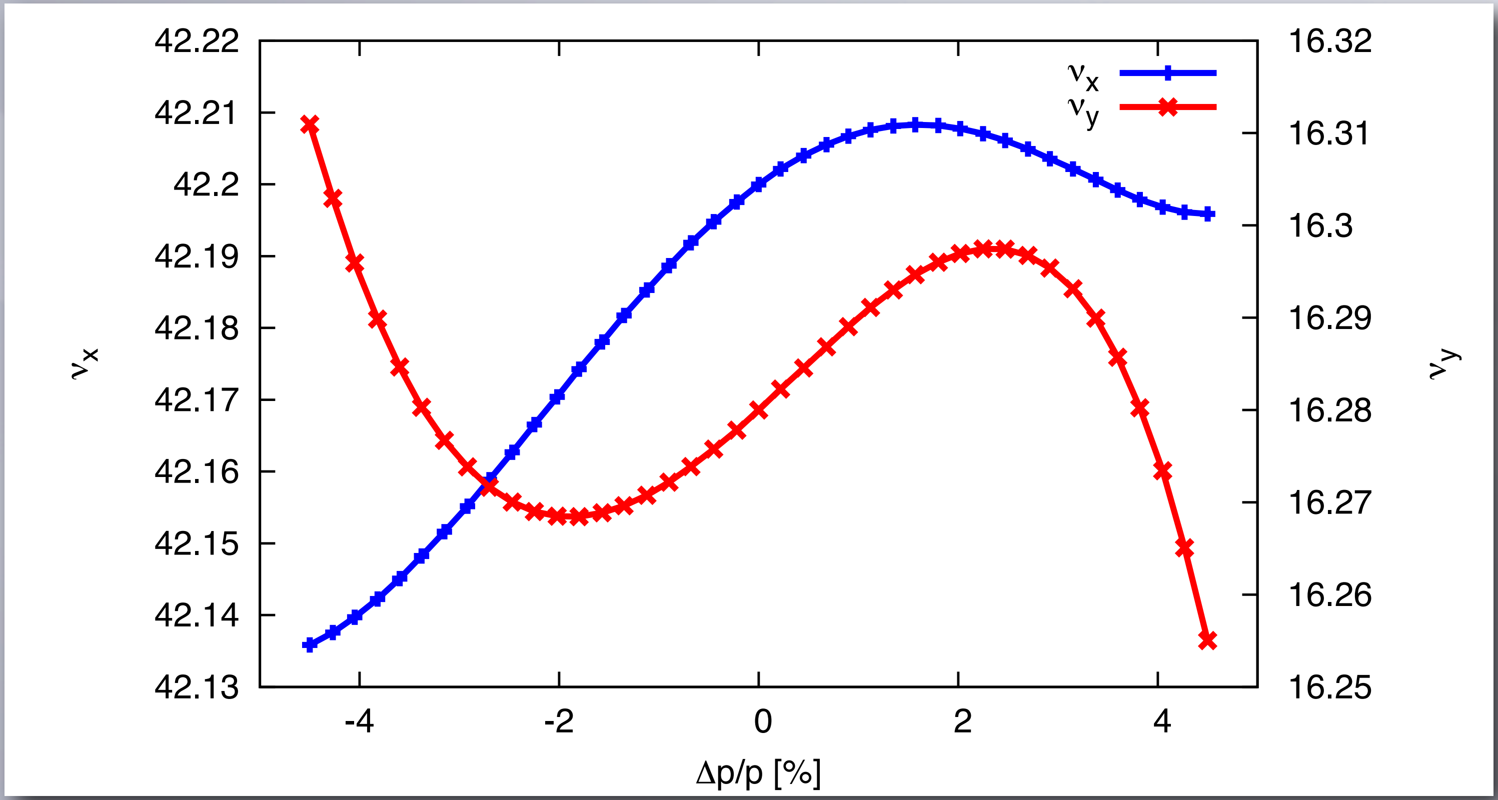
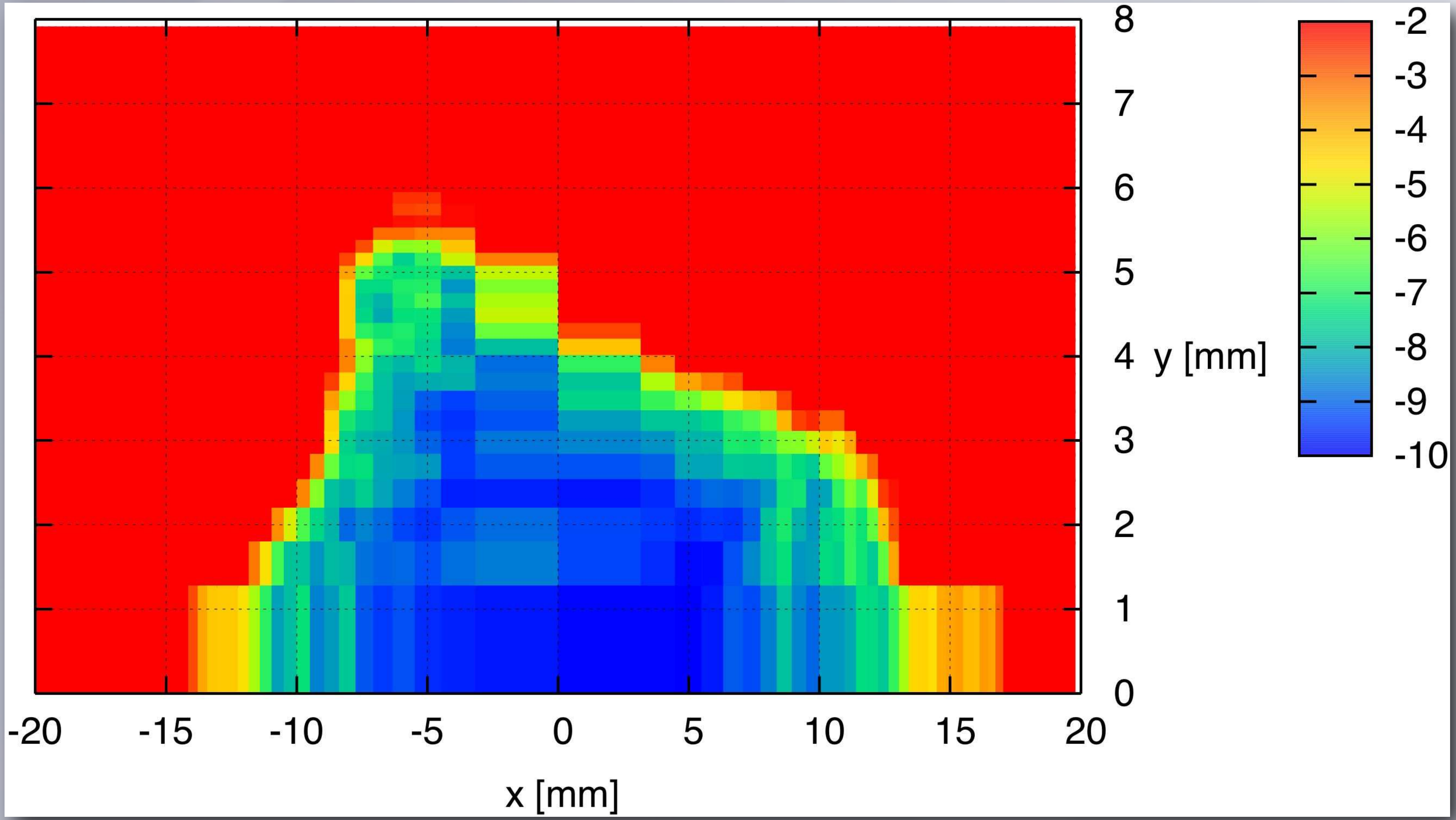
- Chromatic octupoles/decapoles not required to further reduce chrom. footprint



Nonlinear Optimization of the MAX IV 3 GeV SR

Results of nonlinear optimization with octupoles in MAX IV

- ADTS limited → very compact tune footprint → large on-momentum DA



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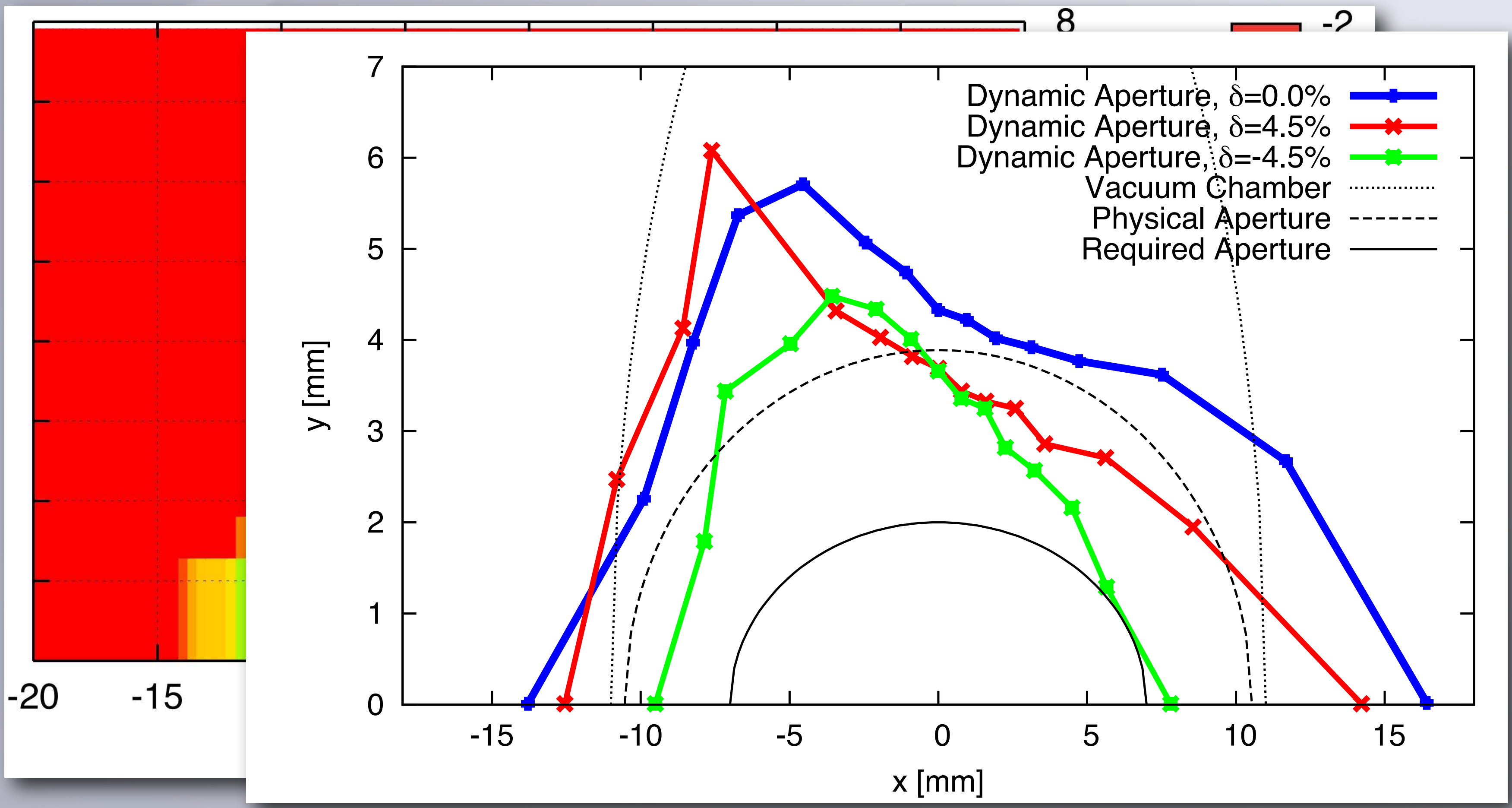

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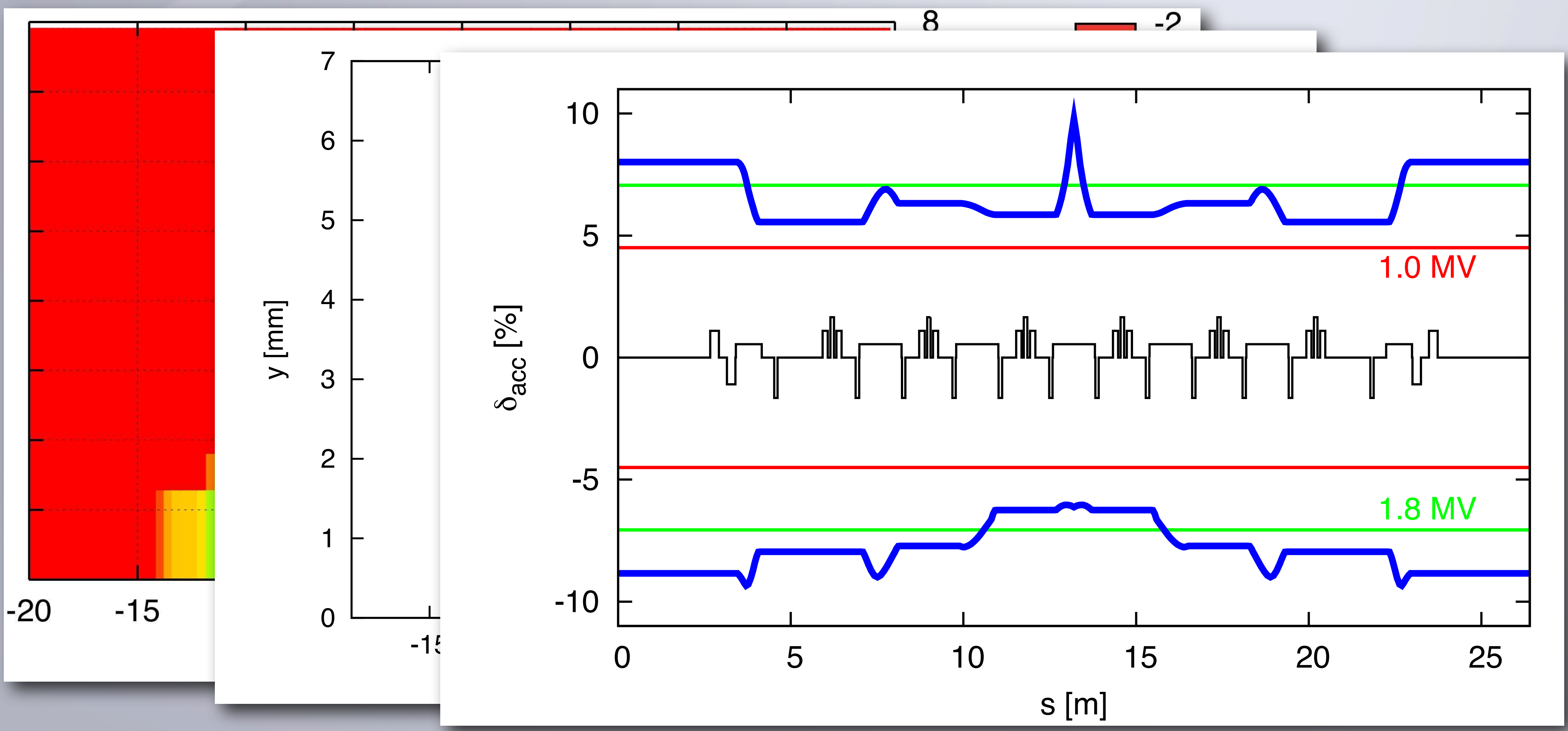

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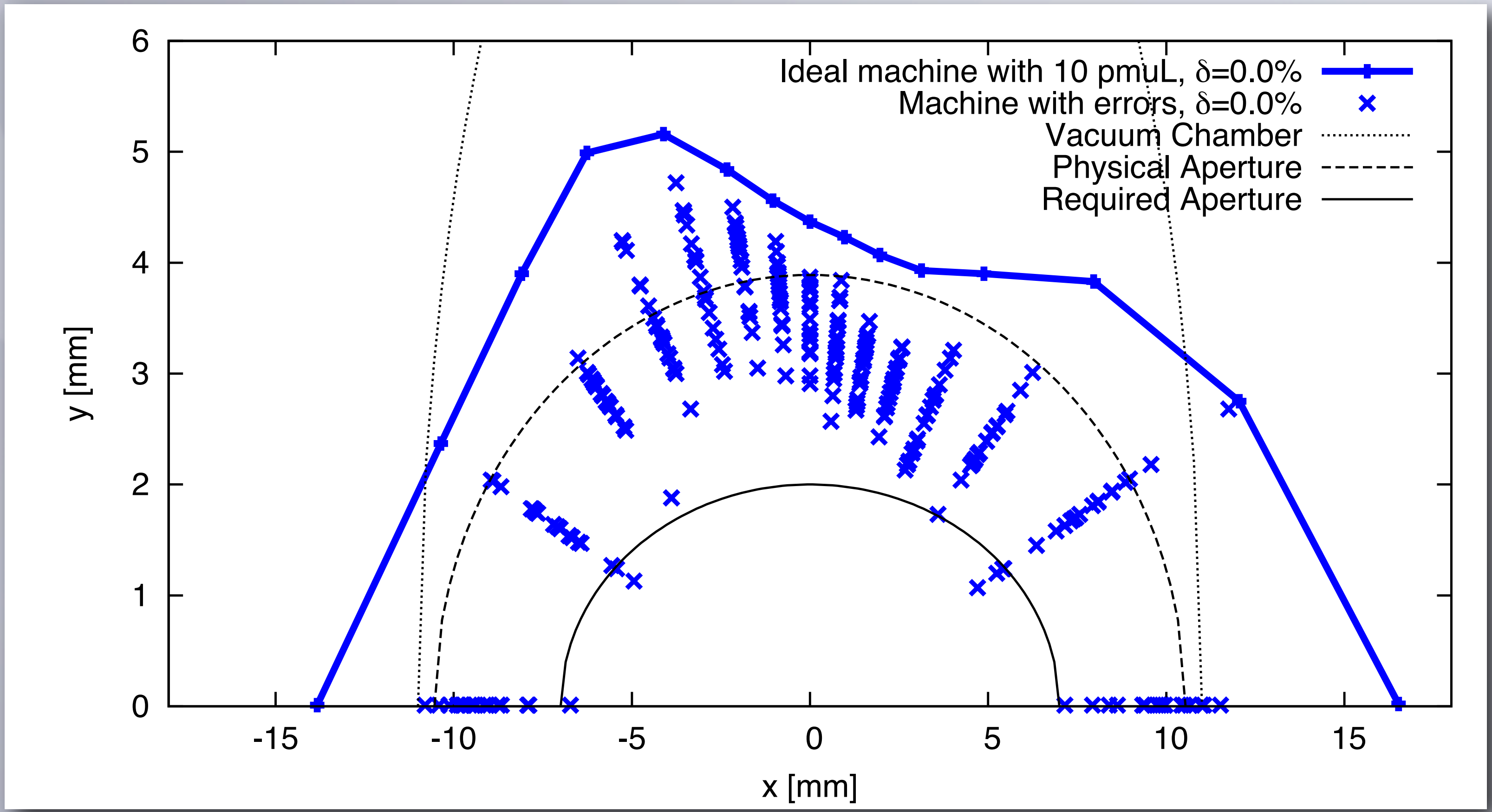
- Tracking with errors and IDs confirms DA remains sufficient



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Along with other ingredients...

- 100 MHz RF system with Landau cavities (300 MHz)
  ➡ Increase Touschek lifetime & limit IBS emittance blowup
- As emittance decreases (DWs, strong IDs) → Touschek lifetime increases

Nonlinear Optimization of the MAX IV 3 GeV SR

Results

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