Proposal for a High Precision Energy Calibration of the MAX II Electron Beam Using Resonant Spin Depolarization

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Abstract

In a perfectly flat storage ring electrons polarize antiparallel to the main bending magnet field due to the emission of spin-flip radiation. A fast kicker magnet can be used for resonant excitation of the beam at the spin precession frequency thus depolarizing the beam. This frequency happens to be proportional to the average beam energy with a factor which only depends on the g-factor of the electron. The polarization level influences the beam lifetime due to the polarization-dependent Touschek scattering process. A sudden change of polarization gives rise to a change of loss rates. Thus a fast vertical kicker magnet driven by a frequency generator and a fast loss monitor are sufficient to carry out the energy calibration measurement. This note will demonstrate how resonant spin depolarization could be used to calibrate the MAX II electron beam energy drawing from experience gained at SLS [1, 2].

1 Polarization

1.1 Spin-flip Radiation and Polarization Buildup

Electrons gradually polarize while orbiting in storage rings due to emission of spin-flip synchrotron radiation [3, 4, 5]. While being radially accelerated, electrons emit synchrotron radiation in quanta of photons which carry a spin. Therefore two cases must be distinguished: After emitting the synchrotron photon the electron spin stays in its initial state or flips over. It has been shown that only an extremely small fraction ($\approx 10^{-11}$) of the emitted power is due to spin-flip radiation while the large fraction of other synchrotron emissions has no inuence on the electron's spin; polarization is therefore a slow process compared to orbital motion. The transition rates for the two possible spin-flips have been calculated to be:

$$W_{\downarrow\uparrow,\uparrow\downarrow} = \frac{5\sqrt{3}}{16} \cdot \frac{e^2\gamma^5\hbar}{{m_e}^2c^2\rho^3} \cdot \left(1 \pm \frac{8}{5\sqrt{3}}\right) \tag{1}$$

where ρ is the instantaneous bending angle, $\downarrow\uparrow$ denotes a spin-flip leading to a spin antiparallel to the guiding dipole field, and $\uparrow\downarrow$ denotes a spin-flip leading to a spin parallel to the guiding dipole field. The strong imbalance between these transition rates leads to a high level of net polarization, the so-called Sokolev-Ternov level:

$$P_{\mathrm{ST}} = \frac{W_{\downarrow\uparrow} - W_{\uparrow\downarrow}}{W_{\downarrow\uparrow} + W_{\uparrow\downarrow}} = \frac{8}{5\sqrt{3}} = 92.38\%. \tag{2}$$

It can be shown that the polarization buildup of an initially unpolarized beam is then described by:

$$P_p(t) = P_{\mathrm{ST}} \left[1 - \exp\left(-\frac{t}{\tau_p} \right) \right] \tag{3}$$

where τ_p is the time constant of the exponential build-up process:

$$\tau_p = (W_{\uparrow\downarrow} + W_{\downarrow\uparrow})^{-1} \cdot \left(\frac{R}{\rho} \right) \tag{4}$$

$$= 98.66 \cdot \frac{\rho^2 R}{E[\mathrm{GeV}]^5} \tag{5}$$

where $R = (2\pi)^{-1} \oint ds$ is the average machine radius and the scaling factor on the right hand side of (4) takes into account that radiation is emitted mainly in the bends. For the MAX II storage ring with $E = 1.5$ GeV, $\rho = 3.33$ m, and $2\pi R = 90$ m the buildup time is 2063 s which is fairly similar to the buildup time at SLS. However, the MAX III storage ring with $E = 700$ MeV, $\rho = 1.59$ m, and $2\pi R = 36$ m has a very large buildup time of 142 min due to its low energy. Since this buildup time is comparable to the beam lifetime, resonant depolarization is not a suitable energy measurement method for MAX III.

1.2 Spin Precession and Spin Tune

The electron spin $\vec{S}$ interacts with the electromagnetic field through the magnetic moment associated with the spin $\vec{\mu}$. In a flat machine the ideal electron sees (apart from accelerating electric fields) only the guiding dipole field $\vec{B} = \vec{B}_\perp$ leading to Thomas precession of the spin vector:

$$\frac{d\vec{S}}{ds} = \vec{\Omega}_{\mathrm{sp}} \times \vec{S} \tag{6}$$

$$\vec{\Omega}_{\mathrm{sp}} = \underbrace{\frac{e\vec{B}_\perp}{m_e c\gamma}}_{\vec{\omega_0}} \cdot a\gamma \tag{7}$$

where a is the anomalous magnetic moment of the electron:

$$a = \frac{g-2}{2} = 0.00115965218. \tag{8}$$

Upon recognizing the relativistic cyclotron frequency (the revolution frequency of the electrons in the storage ring ω_0) in (7), the equation can easily be rewritten as

$$\vec{\Omega}_{\mathrm{sp}} = \vec{\omega_0} \cdot \nu_{\mathrm{st}} \tag{9}$$

where $\nu_{\mathrm{st}} = a\gamma$ is the so-called spin tune. It describes the number of times the spin vector precesses around the guiding dipole field vector for one revolution of the particle around the ring. At the design energy of 1.5 GeV the MAX II ring has a spin tune of 3.405. The anomalous magnetic moment of the electron (8) has been determined with great precision. The simple relationship between spin precession frequency and electron energy (9) can therefore be used to determine the precise energy of the electrons in a storage ring.

1.3 Depolarizing Effects and Equilibrium Polarization

A real storage ring is not perfectly flat and the electrons performing betatron oscillations around the closed orbit are under the influence of more than just the main dipole field. Horizontal magnetic field components $B_{\parallel}$ alter the precession of the spin vector around the guiding dipole field vector thus changing the vertical spin component $S_{\perp}$. In the event that $S_{\perp} = 0$ the beam is depolarized. In analogy to the beam emittance (an equilibrium between radiation damping and quantum excitation), the effective beam polarization is an equilibrium between the polarizing spin-flip radiation and spin diffusion due to depolarizing effects. It can be shown that depolarizing effects lead to an exponential decay of polarization:

$$P_d(t) = P_{\mathrm{ST}} \exp\left(-\frac{t}{\tau_d}\right) \tag{10}$$

and therefore the buildup to equilibrium polarization is described by:

$$P(t) = P_{\mathrm{eff}}\left(1 - \exp\left(-\frac{t}{\tau_{\mathrm{eff}}}\right)\right) \tag{11}$$

with the effective polarization level:

$$P_{\mathrm{eff}} = P_{\mathrm{ST}} \frac{\tau_d}{\tau_p + \tau_d} \tag{12}$$

and the effective polarization buildup time:

$$\frac{1}{\tau_{\mathrm{eff}}} = \frac{1}{\tau_p} + \frac{1}{\tau_d} \tag{13}$$

Ideally depolarizing effects are small and hence the characteristic depolarizing time τ_d is large. According to (12) this then leads to equilibrium polarization values close to the theoretical maximum of $P_{\mathrm{ST}} = 92.4\%$. In order to reach polarization levels above 60% the depolarizing time needs to be at least twice as large as the polarization buildup time. At SLS effective polarization levels in excess of 80% were observed.

The energy calibration measurement requires high effective polarization levels. It is therefore imperative to reduce all contributions to depolarization: effectively this means minimizing all horizontal magnetic fields. Apart from proper machine alignment (reducing dipole pitch and roll as well as any misaligned quadrupole and sextupole magnets) this implies a flat orbit (no vertical kicks) and going centered through all quadrupoles (no bumps). It is however also important to make sure there is no mean corrector strength in the dispersive sections as this would lead to an effective shift of the closed orbit and hence to a different beam energy. Finally, it is also important to verify that the betatron motion of the beam does not enhance depolarization (resonance condition) which could prevent high effective polarization levels. The betatron tunes at full energy in MAX II are $\nu_x = 9.23$ and $\nu_y = 3.18$ [6]. Their fractional parts are far away from the fractional part of the spin tune at 0.405 and therefore this should not be a problem.

2 Energy Calibration

Precise energy calibration by means of resonant depolarization has been successfully carried out at several electron storage rings. Examples are BESSY II [7], ALS [8], and SLS [1, 2]. This note draws mainly from the experience gathered at SLS. It is suggested to carry out the experiment at MAX II in very close accordance with the measurements at SLS.

2.1 Polarization Measurement

In a first step the electron beam is injected into the machine, the energy ramped, and the orbit corrected according to the requirements specified in section 1.3. From then on the machine should then be kept as "quiet" as possible (kickers off, no further orbit correction, no ID gap movement, etc.) to ensure maximum polarization buildup. After the buildup, a high level of equilibrium polarization has to be measured in order to insure that the beam can subsequently be depolarized. One method to do this is the use of a dedicated Compton polarimeter which measures the abolute beam polarization at any time. Another method which does not require a dedicated polarimeter makes use of the fact that the cross section for Touschek scattering is polarization dependent:

$$\sigma_{ts} = f_1(\beta, \Theta) - P^2 \cdot f_2(\beta, \Theta, \Phi) \tag{14}$$

where f_i are functions of β and the scattering angles Θ and Φ. Increasing beam polarization P leads to a smaller Touschek cross section and therefore to less losses of beam particles. On the other hand, a sudden decrease in beam polarization will lead to a rise of Touschek scattering losses. Therefore changes of Touschek loss rates can be correlated with changes in the polarization level of the beam. The product of beam current and Touschek lifetime $I\tau_{\mathrm{ts}}$ is usually a constant. Changes in the Touschek scattering rate will however lead to changes of this product. The equilibrium polarization level P_{eff} can therefore be determined by observing $I\tau_{\mathrm{ts}}$ during polarization buildup, fitting from it the buildup time τ_{eff}, and applying (12),(13). The disadvantage of this method is that it does not give the beam polarization at arbitrary times; it can only reveal the equilibrium polarization after the buildup has been observed (this can easily take two buildup times, i.e. over an hour for MAX II).

Finally, since this method makes use of the polarization dependence of Touschek scattering, beam lifetime should be dominated by the Touschek lifetime. According to [9] the MAX II beam at 180 mA current is indeed Touschek-limited:

$$\frac{1}{\tau} = \frac{1}{\tau_{ts}} + \frac{1}{\tau_{el}} + \frac{1}{\tau_{bs}} = (23.8\ \mathrm{h})^{-1} \tag{15}$$

with Touschek scattering lifetime $\tau_{\mathrm{ts}} = 37$ h, elastic scattering lifetime $\tau_{\mathrm{el}} = 134$ h, and inelastic scattering (bremsstrahlung) lifetime $\tau_{\mathrm{bs}} = 132$ h. By subtracting other contributions from the measured total beam lifetime τ, the Touschek lifetime τ_{ts} can be extracted and the product $I\tau_{\mathrm{ts}}$ determined.

2.2 Resonant Depolarization

Once a high equilibrium polarization has been verified, the beam can be depolarized by applying time-varying horizontal magnetic fields in resonance with the spin precession (a vertical kicker magnet is ideal). Keeping in mind (9), the spin precession frequency can be split up:

$$\Omega_{\mathrm{sp}} = \omega_0 \cdot \nu_{\mathrm{st}} = \omega_0 \cdot a\gamma = \underbrace{\omega_0 \cdot \lceil a\gamma \rceil}_{\text{integer multiple of } \omega_0} + \underbrace{\omega_0 \cdot (a\gamma - \lceil a\gamma \rceil)}_{\text{depolarizing excitation}} \tag{16}$$

where $\lceil a\gamma \rceil$ denotes the integer part of the spin tune. The depolarizing frequency f_d is therefore given by:

$$f_d = f_0 \cdot (a\gamma - \lceil a\gamma \rceil) \tag{17}$$

where f_0 is the revolution frequency of the electrons in the storage ring. For MAX II with a fractional spin tune of 0.405 (at design energy) this corresponds to a resonant depolarizing frequency of 1.351 MHz. Finally, the kicker magnet strength required to resonantly depolarize the beam has to be determined. While passing the kicker magnet the electron spins obey to the simple equation:

$$\frac{d\vec{S}}{ds} = \vec{\Omega}_{\mathrm{kick}} \times \vec{S} = \begin{pmatrix} 0 \\ -\frac{eB_{\mathrm{kick}}}{m_e c} a \\ 0 \end{pmatrix} \times \vec{S}. \tag{18}$$

Therefore, when an electron passes the kicker magnet its spin precession cone angle is widened by:

$$\Delta\theta = \frac{e\,a}{m_e c} \cdot (B_{\mathrm{kick}} \cdot \Delta l) \tag{19}$$

where Δl is the length of the kicker magnet. In order to open the cone angle to $\pi/2$ (leading to $S_\perp = 0$) the following depolarizing time is then required:

$$\tau_{\mathrm{depol}} = \left(\frac{\pi}{2}\right)^2 \frac{1}{f_0} \frac{m_e c}{e\,a} \cdot \frac{1}{(B\Delta l)_{\mathrm{kick}}} \tag{20}$$

where a sinusoidal kicker waveform should been assumed. For practical reasons this depolarizing time should be kept below 1 s. The required kicker strength is therefore:

$$(B\Delta l)_{\mathrm{kick}} > 1.087 \times 10^{-6}\,\mathrm{T\,m}. \tag{21}$$

Depending on the dwell setting of the frequency generator the kicker amplifier power can be varied. At SLS successful runs were conducted with the following settings: 50 W at 2 Hz/s dwell, 135 W at 10 Hz/s dwell, and 250 W at 18 Hz/s dwell.

2.3 Energy Calibration Measurement

For the energy calibration measurement the frequency which is fed to the kicker magnet is swept over a predefined range. When the resonant depolarizing frequency is reached, the beam depolarizes. The observer carrying out the experiment recognizes the increase of the loss rate and/or the sudden drop in $I\tau_{\mathrm{ts}}$. The applied excitation frequency is noted which then renders the beam energy according to (17). The frequency sweep range is therefore determined by the energy range to be

scanned. A first scan at MAX II could be $\pm 1\%$ around the design energy. This corresponds to sweeping the excitation frequency from 1.2373 MHz (1.485 GeV) to 1.4642 MHz (1.515 GeV). If the dwell for this 227 kHz sweep is set to 100 Hz/s (a coarse setting, but good for the initial wide range sweep) the time required for the sweep is 38 min which, taking into account the overall beam lifetime, is practical.

In order to measure the sudden increase of Touschek scattered particles at the depolarizing resonance the product $I\tau_{\mathrm{ts}}$ can be monitored. Apart from polarization, this product should remain constant for the duration of the beam lifetime. It will rise due to polarization buildup after injection and it will decrease immediately when the depolarizing resonance is hit (at SLS this was a 10% effect). Another method to identify the depolarizing resonance is by detecting the actual Touschek scattered electron pairs; this can be accomplished with a pair of scintillators set up in coincidence. If this event rate is normalized with the beam current squared $\dot{N}_{\mathrm{ts}}/I^2$ it should remain constant for the duration of the beam lifetime. Once the depolarizing resonance is hit, this rate will however show an immediate increase. There are different approaches for the location of the scinitillators:

- At BESSY II [7] and SPEAR [10] the scintillators were installed downstream of a high-dispersion area. One of the scintillators was placed inside the ring, the other outside. The idea behind this setup is to measure the Touschek scattered electrons as their dispersive orbit is no longer within the acceptance of the machine. The problem with this approach is that inelastically scattered electrons (energy below machine acceptance) will also be detected by the scintillator inside the ring and hence generate a lot of background.
- At ALS [8] and SLS [1, 2] the scintillator pair was installed downstream of a narrow-gap in-vacuum insertion device with a closed gap. This narrow gap presents an global aperture limitation and makes it the ideal location to collect Touschek scattered pairs. At SLS the sudden loss rate increase when the depolarizing resonance was hit was a very clear effect (60% loss rate increase).

Since the scintillators detect photons from bremsstrahlung emitted when the Touschek scattered electrons hit the surrounding chamber it is suitable to use crystal scintillators (for example NaI).

Finally, once the depolarizing resonance has been detected two important checks need to be made. The first check is to make sure that the applied excitation frequency corresponds to the actual beam energy and not to the mirror energy. Because of the Nyquist theorem the resonance frequency has to be assigned to the fractional part of the spin tune on both sides of the half-integer. For MAX II with a fractional spin tune of 0.405 this means there exists a mirror frequency corresponding to a fractional spin tune of 0.095. This ambiguity is depicted in Fig. 1. A simple method to determine which beam energy the resonance frequency corresponds to, is to slightly vary the main RF frequency and then repeat the measurement. Above transition energy an increase of the RF frequency will lead to an energy decrease and hence the resonance frequency has to decrease as well; if the recorded frequency instead rises, it is the mirror frequency.

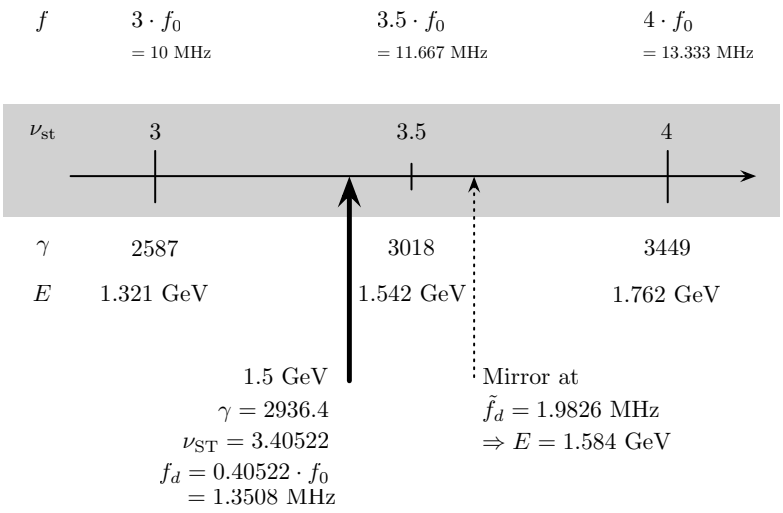

Figure 1: *Spin tunes ν_{st} with corresponding spin precession frequencies f and beam energies E. The spin tune for the MAX II design energy of 1.5 GeV is indicated. Due to the Nyquist theorem the depolarizing frequeny f_d has a mirror frequency $\tilde{f}_d$ which corresponds to a mirror energy of 1.584 GeV.*

The second check is to distinguish the main depolarizing resonance from its sidebands. Due to synchrotron oscillations within the electron bunch the beam energy is not sharp. It can be shown that as a consequence the spin tune has sidebands at integer multiples of the synchrotron tune:

$$f_d = f_0 \cdot \nu_{\mathrm{st}} \pm n \cdot Q_s, \quad n \in \mathbb{N}_0. \tag{22}$$

At MAX II with a detuned Landau cavity the synchrotron sidebands will be separated from the main resonance by $7 - 8$ kHz (corresponding to a difference in beam energy of roughly 1 MeV). This separation is sufficient to distinguish the main resonance from its sidebands. However, for the energy calibration it is still necessary to identify the main depolarizing resonance from within several resonance candidates. This can be accomplished by slightly changing the main RF voltage and then repeating the measurement. Due to the dependence of the synchrotron tune on RF voltage $Q_s \propto \sqrt{V_{\mathrm{rf}}}$ a shift of the RF voltage will change the distance between the main resonance and its sidebands. When comparing the $I\tau_{\mathrm{ts}}$ or loss rate data from before and after the RF voltage shift, one candidate will remain unchanged while the others will have shifted. The unchanged candidate corresponds to the main depolarizing resonance; the shifted candidates are sidebands.

3 Applications and Benefits for MAX II

An obvious benefit of the proposed energy measurement is the better understanding of the MAX II storage ring. In addition precise beam energy knowledge can be used to calibrate undulator performance (the electron beam energy is a free parameter for

fitting undulator spectra) which is of interest to the beam lines and user community.

Energy calibration using resonant spin depolarization is not a fast measurement that can be done parasitically during regular user operation. However, it determines the beam energy with very high precision. This makes it an ideal method to investigate energy stability over longer periods of time. Together with observations of the main RF frequency such data can be used for example to monitor changes in the storage ring geometry (slab settlement, temperature stability, seasonal variations, etc.).

Finally, due to the high precision of the proposed energy measurement, it is ideal to investigate the momentum compaction factor, in particular the nonlinear contribution. This contribution is very small and can only be experimentally determined if the energy is known with high precision. The momentum compaction can be defined as:

$$-\frac{\Delta f_{\mathrm{rf}}}{f_{\mathrm{rf}}} = \alpha_c \frac{\Delta E}{E} + \alpha_2 \left(\frac{\Delta E}{E}\right)^2 \tag{23}$$

where α_c is the linear momentum compaction factor ($\alpha_c = 3.91 \times 10^{-3}$ at MAX II [6]) and α_2 is the nonlinear contribution. By slightly varying the main RF frequency of the MAX II storage ring and repeating the energy calibration measurement both linear and nonlinear contributions of the momentum compaction can be determined experimentally.

4 Requirements for Implementation at MAX II

- Real-time logging of beam current and beam lifetime with a time stamp (sampling > 1 Hz)
- A frequency generator for the frequency sweeps (adjustable range, step size, and dwell)
- A fast and sufficiently strong vertical kicker magnet with the frequency generator as a source
- A pair of scintillators in coincidence installed at the MAX II storage ring (location according to section 2.3); signal readout with time stamp

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