Energy Calibration With High Precision

- Bending magnet field measurements (Hall Probes): \( \frac{\Delta E}{E} = (1 - 2) \cdot 10^{-3} \)
- For higher precision \( \rightarrow \) Resonant Spin Depolarization
- In a flat machine the ideal electron sees \( \vec{B}_\perp \rightarrow \) spin vector precesses according to Thomas-BMT equation:

\[
\omega_{\text{spin}} = \omega_{\text{rev}} \nu_{ST} = \omega_{\text{rev}} \cdot a \gamma \\
a = \frac{g - 2}{2} = 0.00115965(2)
\]

- SLS: \( E = 2.4 \text{ GeV}, \omega_{\text{rev}} = 1042 \text{ kHz} \)
\( \rightarrow \nu_{ST} = 5.4465 \) corresponding to \( \omega_{\text{res-depol}} \approx 465/577 \text{ kHz} \)
Polarization

- Spin-flip synchrotron radiation (only \( \approx 10^{-11} \) of total radiation power) → polarization build-up anti-parallel to main bending field:
  \[
P(t) = P_0(1 - e^{-\frac{t}{\tau}})
  \]
  \[
  \frac{1}{\tau} = \frac{1}{\tau_p} + \frac{1}{\tau_d}
  \]
  \[
P_0 = P_{ST} \frac{\tau_d}{\tau_p + \tau_d}
  \]
  \(P_{ST} = 92.4\%\) theoretical maximum for a flat ring
  \[
  \tau_p = \left( \frac{5\sqrt{3}}{8} \cdot \frac{e^2 \hbar}{m_e^2 c^2} \right)^{-1} \cdot \frac{P^3}{\gamma^5}
  \]  
  @SLS: \(\tau_p = 1873\) s (\(\gamma = 4700\))
  @LEP: \(\tau_p = 360\) min (\(\gamma = 86000\))

- \(\tau_d \propto (a\gamma)^{-2}\)
- \(\tau_d \gg \tau_p\) for low-energy machines

- How to measure polarization without a polarimeter?
  - Touschek lifetime is polarization dependent: \(\sigma_{ts} = f_1 - P^2 f_2\)
  - Choose filling with high bunch current → \(\tau\) is dominated by \(\tau_{ts}\)
  - Identify changes in \(I \cdot \tau_{ts}\) with changes in \(P\)
For SLS storage ring: $\tau_p = 1873$ s

From fit: $\tau = (1837 \pm 1)$ s $\rightarrow \tau_d = 95.6 \cdot 10^3$ s

$\rightarrow P_0 = P_{ST} \cdot \frac{\tau_d}{\tau_p + \tau_d} = 91\%$
Resonant Depolarization

- Feed sinusoidal signal to vertical tune kicker magnet
- Sweep signal frequency over interval around resonant depolarizing frequency → At \( \omega_{res-depol} \) the ensemble’s mean spin vector can be tilted into the horizontal plane (together with spin diffusion) → Reduce polarization → Reduce \( I \cdot \tau_{ts} \)
- Verification of the resonance through (uncalibrated) loss monitor coincidence signal from two scintillators installed in the vicinity of in-vacuum undulator U24 → Pairs of Touschek-scattered electrons
Energy Calibration

- Froissart-Stora fit for isolated resonance crossing
  \[ E = (2.4361 \pm 0.00018) \text{ GeV} \]
  \[ \frac{\Delta E}{E} \approx 7 \cdot 10^{-5} (10^2 \times \text{higher than Hall probe measurements!}) \]
- Energy is 1.5% higher than determined from dipole calibration
  \[ \text{Confirms previous increase of the quadrupole and sextupole magnet strength of } +1.2\% \text{ with respect to design at } 2.4 \text{ GeV} \]
Energy Calibration

- $E$ is in excellent agreement with the energy derivation from a scan of the 7th harmonic of U24 revealing $E = 2.44(2)\text{ GeV}$
High precision of energy calibration → measure non-linearity of Momentum Compaction:

\[-\frac{\Delta f_{RF}}{f_{RF}} = \alpha \cdot \frac{\Delta E}{E} + \alpha_1 \cdot \left(\frac{\Delta E}{E}\right)^2\]

Results: \(\alpha = 5.6 \cdot 10^{-4}\) and \(\alpha_1 = 4.4 \cdot 10^{-3}\)

- From dispersion measurements: \(\alpha = 6.0 \cdot 10^{-4}, \alpha_1 = 4.2 \cdot 10^{-3}\)
- From machine model and detuning: \(\alpha = 6.5 \cdot 10^{-4}, \alpha_1 = 4.5 \cdot 10^{-3}\)

But only three data points (no error bars on measurement!)

→ Need to repeat measurement and acquire more data!