Prof. Dr. Albin F. Wrulich EPFL, Winter Term 2005/2006

Introduction to Particle Accelerator Physics

Tutorial 2 - Problems

Discussion: 22.11.2005 Hand in: 29.11.2005 Solutions: 6.12.2005

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1. Thin Lens Approximation

Derive the transport matrix for a quadrupole in the thin lens approximation. Consider the Taylor expansion of the functions $\sin(x)$ and $\cos(x)$. Keep in mind the definition of the focal length f.

2. Drift Sections and Quadrupole Doublets

Phosphor screens allow us to measure particle displacements x rather easily. Usually it is more difficult to measure a particle trajectory's angle x'.

a) Assume a drift section with length L. Set up the transport matrix for this drift and show how the initial angle x'_0 can be derived from the displacements x_0 and x.

b) Now assume there is no way to measure the initial displacement, but a quadrupole doublet (focal length f and distance L between the quadrupoles) with properly tuned focusing can be used. Set up the transport matrix for this doublet in the thin lens approximation. Can we now measure the initial angle x'_0?

c) Assume electrons with a kinetic energy of 650 MeV and a distance $L = 931$ mm between the quadrupoles of the doublet. What focusing gradient is now required to measure the initial angle x'_o?

3. Hill's Equation

From the lecture, recall Hill's equation

$$x'' + k(s) \cdot x = 0$$

where the periodic function $k(s)$ is defined for all s of the (circular) accelerator so that $k(s + C) = k(s)$ where C is the circumference of the accelerator.

a) Assume $k(s)$ is constant for the entire accelerator. What does Hill's equation now represent?

b) What is the solution of this equation?

c) Assume now $k(s)$ is a piece-wise constant function. Describe a qualitative solution for this equation.

4. Evolution of a Phase Space Ellipse
Assume an initially elliptical phase space distribution.
a) Make a drawing to illustrate how the phase space ellipse transforms along a drift.
b) Assume an initially diverging beam. Make a drawing to illustrate the phase space evolution as this beam passes a focussing quadrupole.

5. Transformation of the Beta Function
Assume β_0 at the center of a drift section s_0. Assume further that this is a symmetry point where $\beta_0' = 0$.
a) Calculate the functions $\beta(s)$, $\alpha(s)$, and $\gamma(s)$ by using the transformation property of the beta matrix $B_s = MB_0M^T$.
b) Calculate the functions $\beta(s)$, $\alpha(s)$, and $\gamma(s)$ by direct transformation of the Twiss parameters.
c) Take a close look at the solution of $\beta(s)$. What kind of function is $\beta(s)$? Make a drawing for various β_0.

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