Introduction to Particle Accelerator Physics

Tutorial 1 - Solutions

1. Relativistic Particles

a) Recall from the lecture the definition of $\gamma$

$$\gamma = \frac{E_{tot}}{E_0} = \frac{E_0 + E_{kin}}{E_0} = 1 + \frac{E_{kin}}{m_0c^2}$$

The beam is accelerated up to $E_{kin} = 2.4$ GeV, so

$$\gamma = 1 + \frac{E_{kin}}{E_0} = 1 + \frac{2.4 \text{ GeV}}{\frac{511 \text{ keV}}{c^2} \cdot c^2} = 1 + \frac{2.4 \cdot 10^9 \text{ eV}}{511 \cdot 10^3 \text{ eV}} \approx 4698$$

b) $\gamma = \frac{1}{\sqrt{1 - \beta^2}} \implies \beta = \sqrt{1 - \gamma^{-2}} = 0.9999999733... \approx 1$

$\implies$ The electrons are highly relativistic: $v = \beta c \approx c$

c) $f_{rev} = \frac{v}{C} = \frac{\beta c}{C} \approx \frac{c}{C} = \frac{3 \cdot 10^8 \text{ m/s}}{288 \text{ m}} = 1.042 \text{ MHz}$

d) $m_e = 511 \frac{\text{keV}}{c^2} = 511 \cdot 10^3 \text{ V} \cdot 1.602 \cdot 10^{-19} \text{ C} \cdot \frac{1}{(3 \cdot 10^8 \text{ m/s})^2} = 9.11 \cdot 10^{-31} \text{ kg} = m_e$

e) $\gamma = \frac{E_{tot}}{E_0} = \frac{E_0 + E_{kin}}{E_0} = 1 + \frac{E_{kin}}{m_0c^2}$

At SLS electrons are accelerated to 2.4 GeV: $\gamma_{SLS} = 1 + \frac{2.4 \text{ GeV}}{\gamma_{31 \text{ keV}}} \approx 4698$

At LHC protons are accelerated to 7 TeV: $\gamma_{LHC} = 1 + \frac{7 \text{ TeV}}{\gamma_{938 \text{ MeV}}} \approx 7464$

$\implies \gamma_{LHC} : \gamma_{SLS} = 1.589$
2. Dipole Magnets vs. Static Electric Fields
Highly relativistic electrons: $v \approx c$. Recall the Lorentz force in absolute values: $F = q(E + vB)$. Using a static electric field: $F_E = qE$, but using a dipole magnet: $F_B = qvB \approx qcB$. By using dipole magnets the force applied to the particle is scaled by the particle’s velocity, thus much less field strength is required to apply the same force as a static electric field.

3. Cyclotron
At injection the energy is $72$ MeV: $\gamma_i = 1 + \frac{72}{938}$ MeV.
At extraction the energy is $590$ MeV: $\gamma_e = 1 + \frac{590}{938}$ MeV
Using the magnetic rigidity and remembering that $\gamma \beta = \sqrt{\gamma^2 - 1}$ we can derive

$$B\rho = \frac{p}{e} = \frac{m_0\gamma\beta c}{e}$$
$$\rho \propto \gamma \beta$$
$$\implies \rho \propto \sqrt{\gamma^2 - 1}$$
So the orbit radius at extraction has increased by the factor

$$\frac{\rho_e}{\rho_i} = \frac{\sqrt{\gamma^2_e - 1}}{\sqrt{\gamma^2_i - 1}} = 3.22$$
If this machine were to be operated as an iso-cyclotron, this would require

$$\frac{B_e}{B_i} = \frac{\gamma_e}{\gamma_i} = 1.51$$

4. Electron Beam in a Storage Ring
a) Recall from the lecture the definition of the magnetic rigidity

$$B\rho = \frac{p}{e}$$
In more practical units this becomes

$$B\rho \ [T \cdot m] = \frac{1}{0.29979} \cdot p \ [GeV/c]$$
Here we assume the whole storage ring consists only of bending magnets, so $R = \frac{C}{2\pi}$ and we get

$$BR \ [T \cdot m] = \frac{1}{0.29979} \cdot p \ [GeV/c] \implies B \ [T] = \frac{1}{0.29979} \cdot p \ [GeV/c] \cdot \frac{2\pi}{C \ [m]} = 175 \ mT$$
b) 
\[ B \rho [\text{T} \cdot \text{m}] = \frac{1}{0.29979} \cdot p [\text{GeV/c}] \implies B [\text{T}] = \frac{1}{0.29979} \cdot p [\text{GeV/c}] \cdot \frac{1}{\rho [\text{m}]} = 1.334 \text{ T} \]

c) The earth’s magnetic field will deflect the particles in the beam if it has a non-zero transverse (i.e. vertical or radial) component with respect to the particle’s orbit. However, the strength of the earth’s magnetic field (estimated to be \(3 \cdot 10^{-5} \text{ T}\)) is far less than the bending magnet strength which is roughly 1 T. The tolerance of the magnets will be roughly 0.1% and therefore there will be corrector coils capable of correcting deviations of this order. Thus, we do not have to worry about the earth’s magnetic field.

d) 
\[ F_{\text{grav}} = m_e g = 9.109 \cdot 10^{-31} \text{ kg} \cdot 9.81 \text{ m/s}^2 = 8.94 \cdot 10^{-30} \text{ N} \]
\[ F_{\text{bend}} = e v B = e \beta c B \approx e c B \approx 1.602 \cdot 10^{-19} \text{ C} \cdot 3 \cdot 10^8 \text{ m/s} \cdot 1.334 \text{ T} = 6.41 \cdot 10^{-11} \text{ N} \]
\[ \implies F_{\text{bend}} \approx 10^{19} \cdot F_{\text{grav}} \]