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Introduction to Particle Accelerator Physics

Solutions to Exercise 3

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1. Dispersion Function
From the lecture recall the general solution of the dispersion function:

$$D(s) = D_0 \cos \frac{s}{\rho} + D_0' \rho \sin \frac{s}{\rho} + \rho \left(1 - \cos \frac{s}{\rho}\right)$$

We can rewrite this and do a Taylor expansion of the sine and cosine functions keeping our limit $\rho \to \infty$ in mind:

$$\begin{aligned} D(s) &= D_0 \cos \frac{s}{\rho} + \rho - \rho \cos \frac{s}{\rho} + D_0' \rho \sin \frac{s}{\rho} \\ &= D_0 \left(1 - \frac{s^2}{2\rho^2} + - \ldots\right) + \rho \left(1 - 1 + \frac{s^2}{2\rho^2} - + \ldots\right) + D_0' \rho \left(\frac{s}{\rho} - \frac{s^3}{3! \cdot \rho^3} + - \ldots\right) \\ &= D_0 \left(1 - \mathcal{O}(\rho^{-2})\right) + \mathcal{O}(\rho^{-1}) + D_0' s + D_0' \mathcal{O}(\rho^{-2}) \end{aligned}$$

And finally for a straight section we apply the limit $\rho \to \infty$:

$$\lim_{\rho \to \infty} D(s) = D_0 + D_0' s$$

So if we enter a straight section with no dispersion at all ($D_0 = D_0' = 0$), $D(s)$ will remain zero. In order to generate dispersion, we have to include a bend in our accelerator optics where $\rho \neq 0$.

2. Momentum Compaction Factor
From the lecture recall the definition of the momentum compaction factor:

$$\frac{\Delta L}{L} = \alpha_c \cdot \frac{\Delta p}{p}$$

From this we can derive the maximum absolute change in path length:

$$\Delta L = \alpha_c \cdot \frac{\Delta p}{p} \cdot L = \alpha_c \cdot \frac{\Delta p}{p} \cdot \frac{c}{f} = 6.3 \cdot 10^{-4} \cdot 9 \cdot 10^{-4} \cdot \frac{3 \cdot 10^8 \text{ m/s}}{1.04167 \cdot 10^6 \text{ Hz}} = 163\ \mu\text{m}$$

Compared to the revolution period of 288m this is a very small path length variation.

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