# Introduction to Particle Accelerator Physics

## Exercise 4

Discussion: 14.12.2004	Hand in: 21.12.2004	Solutions: 4.1.2005

#### 1. Phase Space Representations of Particle Sources

a) Consider a source at  $s_0$  with radius w emitting particles. Make a drawing of this setup in configuration space and in phase space. Which part of phase space can be occupied by the emitted particles?

b) Any real beam emerging from a source like the one above will be clipped by aperture limitations of the vacuum chamber. We can model this by assuming that a distance d away from the source there is an iris with an opening with radius R = w. Make a drawing of this setup in configuration and phase space. Show which parts of phase space are occupied by the beam at a location after the iris.

### 2. Quadrupole Errors and Tune Shifts

In the lecture a quadrupole focussing error was introduced into Hill's equation by replacing the focussing function K(s) with  $K(s) + \Delta k(s)$ . It was then proposed that the error in the focussing function could be represented as a gradient error  $\Delta(kl)$  leading to a tune shift  $\Delta Q = \frac{1}{4\pi}\beta_0\Delta(kl)$ . Assume that the tune shift is small with respect to the tune and prove this statement.

#### 3. Momentum Compaction and Transition Energy

Dispersion leads to path length changes for off-momentum particles. This is characterized by the momentum compaction factor  $\alpha_c$  defined in the lecture. Considering that timing is very important in an accelerator, can path length changes be related to a change of the revolution period? Or in mathematical terms, how does  $\frac{\Delta T}{T}$  depend on  $\frac{\Delta p}{p}$ ? Is it possible to build an accelerator where  $\Delta T = 0$  regardless of  $\Delta p$ ?