

Introduction to Particle Accelerator Physics

Exercise 2

Discussion: 9.11.2004

Hand in: 16.11.2004

Solutions: 23.11.2004

1. Thin Lens Approximation

Derive the transport matrix for a quadrupole in the thin lens approximation. Consider the Taylor expansions of $\sin(x)$ and $\cos(x)$. Keep in mind the definition of the focal length f .

2. Drift Sections and Quadrupole Doublets

Phosphor screens allow us to measure particle displacements x rather easily. Usually it is more difficult to measure a particle trajectory's angle x' .

a) Assume a drift section with length L . Set up the transport matrix for this drift and show how the initial angle x'_0 can be derived from the displacements x_0 and x .

b) Now assume there is no way to measure the initial displacement, but a quadrupole doublet (focal length f and distance L between the quadrupoles) with properly tuned focusing can be used. Set up the transport matrix for this doublet in the thin lens approximation. Can we now measure the initial angle x'_0 ?

c) Assume electrons with a kinetic energy of 650 MeV and a distance $L = 931$ mm between the quadrupoles of the doublet. What focusing gradient is now required to measure the initial angle x'_0 ?

3. Stability Criterion for a Circular Accelerator

Recall from the lecture the generalized one-turn transformation matrix M_{Rev} .

a) Without loss of generality we can choose a starting point in the accelerator where $\alpha_0 = -\frac{\beta'_0}{2} = 0$. What does the matrix look like now?

b) Recall the betatron tune defined as $Q = \frac{\mu}{2\pi}$. Think about the relationship between this matrix and phase space. What does μ represent? What does Q represent?

c) Calculate the trace of this matrix. Considering that the cos function has a co-domain of $[-1, +1]$ in \mathbb{R} , can you derive a simple stability criterion for M_{Rev} ?

4. Transformation of the Beta Function

Assume β_0 at the center of a drift section s_0 . Assume further that this is a symmetry point where $\beta'_0 = 0$.

a) Calculate the functions $\beta(s)$, $\alpha(s)$, and $\gamma(s)$ by using the transformation property of the beta matrix $B_s = MB_0M^T$.

b) Calculate the functions $\beta(s)$, $\alpha(s)$, and $\gamma(s)$ by direct transformation of the Twiss parameters.

c) Take a close look at the solution of $\beta(s)$. What kind of function is $\beta(s)$? Make a drawing for various β_0 .

5. Evolution of a Phase Space Ellipse

Assume an initially elliptical phase space distribution.

a) Make a drawing to illustrate how the phase space ellipse transforms along a drift section.

b) Assume an initially diverging beam. Make a drawing to illustrate the phase space evolution as this beam passes a focussing quadrupole.