Prof. Dr. Albin F. Wrulich EPFL, WS 2004/2005

Introduction to Particle Accelerator Physics

Exercise 1

Discussion: 2.11.2004 Hand in: 9.11.2004 Solutions: 16.11.2004

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1. Relativistic Particles
Assume an electron storage ring like the one found at the Swiss Light Source SLS. The circumference of this ring is $C = 288$ m. The stored beam has an energy of 2.4 GeV.
a) What is the ideal particle's γ ?
b) What is the ideal particle's β ? Are the electrons relativistic?
c) What is the revolution frequency of the particles in the ring?
d) The *Particle Data Booklet* (can be obtained from `http://pdg.lbl.gov`) claims the mass of the electron is $m_e = 511$ keV/c^2, but it also says the mass of the electron is $m_e = 9.11 \cdot 10^{-31}$ kg. Are both statements correct? Show how one can be derived from the other.
e) In 1a) you calculated γ for an ideal electron in the SLS storage ring. In the LHC protons will be accelerated and brought to collision at 4 interaction points (IP) where the experiments ALICE, ATLAS, CMS, and LHCB are located. The design energy of LHC is 7 TeV. Which machine's particles have the higher velocity?

2. Electron Beam in a Storage Ring
Assume again an electron storage ring like the SLS with a circumference of $C = 288$ m and an energy of 2.4 GeV.
a) Recall the magnetic rigidity $B\rho$ defined in the lecture. Which dipole strength is required to keep the electron beam on a circular orbit if we assume the storage ring consists of dipole magnets only?
b) Now consider a real storage ring that contains bending magnets but also many straight sections. These are required for components like focusing magnets (quadrupoles, sextupoles, etc.), corrector magnets (kickers, pingers), detectors, insertion devices (wigglers, undulators), RF, etc. The radius of the bending magnets is assumed to be $\rho = 6$ m. What bending magnetic field strength is required to keep the particles on a circular orbit?
c) What about the earth's magnetic field (assume a peak strength of roughly $3 \cdot 10^{-4}$ T)? Under which conditions will the particles be deflected?
d) We have not considered gravity yet! Compare the forces acting on an electron in the beam. Which is larger, the gravitational force or the bending magnet force?

3. Dipole Magnets vs. Static Electric Fields
In accelerators, dipole magnets are used to deflect charged particles. The same could be achieved with static electric fields. Why are dipole magnets chosen instead? Assume highly relativistic electrons in a storage ring. What field strengths are necessary in order to apply a certain force on the particles?

4. Hill's Equation
From the lecture, recall Hill's equation

$$x'' + k(s) \cdot x = 0$$

where the periodic function $k(s)$ is defined for all s of the (circular) accelerator so that $k(s + C) = k(s)$ where C is the circumference of the accelerator.

a) Assume $k(s)$ is a constant for the entire accelerator. What does Hill's equation now represent?
b) What is the solution of this equation?
c) $k(s)$ is now assumed to be piece-wise constant. Describe a qualitative solution for this equation.

Assistant: Simon C. Leemann, `simon.leemann@psi.ch`, 056/310 3343